Composite Structures 119 (2015) 578–597

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Free vibration analysis of four-unknown shear deformable functionally graded cylindrical microshells based on the strain gradient elasticity theory

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ARTICLE INFO

Article history:
Available online 28 September 2014

Keywords:
Four-unknown shear deformation theory
Functionally graded cylindrical microshell
Strain gradient elasticity theory
Trapezoidal shape factor

ABSTRACT

In this paper, we present a four-unknown shear deformation theory and then develop a shear deformable functionally graded cylindrical microshell model using the strain gradient elasticity theory. Unlike the existing shear deformable cylindrical shell models, the present one contains four independent displacement functions only and introduces three material length scale parameters. In addition, the trapezoidal shape factor $(1 + z/R)$ of a shell element is taken into account in the expressions of classical and non-classical stresses to obtain the accurate stress-resultants over the thickness. The material properties of the microshell are estimated through the Mori–Tanaka homogenization technique. By using Hamilton's principle, the equations of motion and boundary conditions are obtained. Closed-form solutions are derived for the free vibration problem of cylindrical microshells with simply supported ends and fixed ends respectively. Comparison studies are performed to establish the validity of the derived formulation. Finally, some illustrative examples are presented to investigate the influences of the material length scale parameter, gradient index, thickness-to-radius ratio, thickness-to-length ratio and boundary conditions on the vibration characteristics of cylindrical microshells. Numerical results indicate that both the frequency and higher-order mode shapes exhibit significant size-dependence when the thickness of the microshell approaches to the material length scale parameter.

1. Introduction

Shell structures are one of the most important and commonly used structural elements in engineering applications, such as submarines, aircrafts, missiles, rockets, pipes and storage tanks. A shell structure may be defined as a body enclosed between two closely spaced and curved surfaces. Due to the mechanical and geometric complexities of the shell structures, researchers proposed various shell theories based on different assumptions and simplifications. More detailed descriptions on this subject can be found in previous literature by Leissa [1], Liew [2], Reddy [3], Qatu et al. [4], and Alijani and Amabili [5]. Among various geometrical shapes of shell structures, cylindrical shells are perhaps the most widely studied due to the considerable prevalence in the aerospace and navigation fields. With the increased use of cylindrical shells, a thorough understanding of their vibration characteristics is essential for designers and engineers. A huge amount of research efforts have been devoted to the vibration problem of homogeneous and laminated cylindrical shells (e.g. [6–13]). However, classical composite cylindrical shells always require the designer's effort to tailor different laminate's properties to suit a particular application. Owing to the sharp discontinuity in the material properties at interfaces between two different materials, they may exist stress concentrations which result in sever material failure. To avoid such defects, functionally graded materials (FGMs) were proposed in 1984 by a group of material scientists in Japan, see [14,15], as a means of preparing thermal barrier materials. Typically, FGMs are made from a mixture of metals and ceramics. Due to excellent characteristics of ceramics in heat and corrosive resistances combined with the toughness of metals, the inter-laminar stresses at the interface at the layers would vanish. In the past ten years, the increasing demand to produce lightweight structures has led to the use of FGMs in designing shell structures. The stability and dynamic behaviors of FG cylindrical shell have attracted special attentions of a lot of researchers in the world [16–23].

With the rapid development of technology, micro- and nano-scale structural elements made of homogeneous materials or FGMs have been found extensive applications in microelectronic and micromechanical devices, such as thin films [24], atomic force

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http://dx.doi.org/10.1016/j.compstruct.2014.09.032
0263-8223/© 2014 Published by Elsevier Ltd.

microscopes (AFMS) [25], micro-/nano-electro-mechanical systems (MEMS/NEMS) [26]. In such applications, size effects have been experimentally observed [27–32]. For example, Lam et al. [29] found that the bending rigidity increases about 2.4 times as the beam thickness reduced from 115 μm to 20 μm in the micro-bending testing of epoxy polymeric beams. Using a novel automated torsion balance, Liu et al. [31] conducted torsion tests on polycrystalline copper wires with diameter ranging from 18 μm to 105 μm and observed that the normalized torque at $2a = 18$ μm (a wire radius) is approximately 1.5 times of that for $2a = 105$ μm. It is therefore of prime importance that the size effects resulting from the underling micro-structures is taken into consideration in both theoretical and experimental studies of micro- and nano-scale structures.

It is generally known that the classical continuum theory fails to account for the size effects on the mechanical behaviors. For this reason, various higher-order continuum theories involving additional material length scale parameters have been put forward. As examples of such continuum theories, we can mention classical couple stress theory [33,34], nonlocal elasticity [35], modified couple stress theory [36], strain gradient theory [27,29], etc. The modified couple stress theory [36] and the strain gradient elasticity theory [29] are the two most widely used in modeling and analysis of micro-structural elements. In the former theory, the strain energy density function depends only on the strain tensor and the symmetric part of the curvature tensor. This feature makes the modified couple stress theory easier to use than previous couple stress theory. Henceforth, the modified couple stress theory [36] has been widely accepted and employed to study the size-dependent mechanical behavior of micro- structures such as Bernoulli–Euler microbeam [37], Timoshenko microbeam [38], Reddy beam [39], Kirchhoff microplate [40], Mindlin microplate [41–43], cylindrical microshell [44–46], conical microshell [47]. Compared to the former theory, the latter theory reflects the size effects on micro-structures more comprehensively. It involves three material length scale parameters corresponding to the dilatation gradient vector, the deviatoric stretch gradient tensor and the curvature tensor. It is worth noting that the latter theory can reduce to the former one when the first two material length scale parameters of the latter theory are taken to be zero. Based on the strain gradient elasticity theory, several size-dependent beam, plate and shell models have been developed. For example, Kong et al. [48] and Wang et al. [49] developed strain gradient Bernoulli–Euler and Timoshenko beam models, respectively. These models are used to analyze the static and dynamic characteristics of homogeneous microbeams. According to the above two models, the corresponding finite element methods were respectively proposed by Kahrobaiyan et al. [50] and Zhang et al. [51]. In view of the importance of the effect of nonlinearities, Zhao et al. [52] and Ramezani [53] established strain gradient Bernoulli–Euler and Timoshenko beam models that accounted for geometric nonlinearity, respectively. In the work of Akgöz and Civalek [54], analytical solutions for static bending of Bernoulli–Euler microbeams with various boundary conditions were presented. Li et al. [55] developed a bilayered microbeam model and a bilayered cantilever microbeam subjected to a moment at the free end was solved. To avoid the use of shear correction factor, Akgöz and Civalek [56,57] put forward a strain gradient sinusoidal beam model based on a sinusoidal shear deformation theory. Wang et al. [58] developed a stain gradient Kirchhoff plate model and investigated the static bending, free vibration and buckling problems of simply supported microplates. Later, Movassagh and Mahmoodi [59] found that the double Fourier' series assumed by Wang et al. [58] only satisfy the classical boundary conditions and applied the extended Kantorovich method to solve the resulting governing equations. Ramezani [60] derived the equations of motion and boundary conditions of a general geometrically nonlinear Kirchhoff microplate using the variational method. Zeighampour and Tadi Beni [61] developed a strain gradient cylindrical thin-shell model and studied the free vibration problem of single-walled carbon nanotube with simply supported boundary conditions. It should be noted that the above mentioned works only apply to the homogeneous micro-structures. In recent years, some investigations have been carried out to study the mechanical behavior of functionally graded (FG) micro-structures. For instance, Kahrobaiyan et al. [62] established a size-dependent FG Bernoulli–Euler beam model and assessed the static bending and free vibration behaviors of a simply supported FG microbeam as case studies. Akgöz and Civalek [63] employed the strain gradient elasticity theory to investigate the buckling characteristic of FG Bernoulli–Euler microbeams with different boundary conditions. Ansari et al. [64,65] studied the linear and nonlinear vibration characteristics of strain gradient FG Timoshenko microbeams respectively. Sahmani and Ansari [66] studied the buckling problem of FG microbeam by using a size-dependent beam model including thermal environment effect and higher-order shear deformation effect. It was revealed that temperature change plays more important role in the buckling behavior of FG microbeams with higher values of dimensionless material length scale parameter. Grounded on the work above, Sahmani et al. [67] performed nonlinear free vibration analysis of third-order shear deformable FG microbeams with considering geometric nonlinearity. It was demonstrated that by approaching from metal phase to ceramic phase for a FG microbeam, the linear frequency and nonlinear frequency ratio tend to decrease and increase, respectively. Based on the Reddy shear deformation theory, Sahmani and Ansari [68] studied the free vibration problem of FG microplates and presented the Navier-type solution for simply supported boundary conditions. Zhang et al. [69] presented a novel size-dependent FG curved beam model based on a nth-order shear deformation theory. Based on a trigonometric shear deformation beam theory, Lei et al. [70] set up a new microstructure-dependent trigonometric FG beam model for bending and free vibration of microbeams. More recently, Zhang et al. [71] proposed an improved third-order shear deformable beam model with containing both the elastic foundation and the strain gradient effects for bending, buckling and free vibration problems of FG microbeams embedded in elastic medium. Akgöz and Civalek [72] investigated the longitudinal free vibration of axially FG microbars. The proposed microbar model includes three material length scale parameters which can capture the size effects. Young's modulus, shear modulus, mass density and material length scale parameters of the microbar is assumed to be smoothly varied along the axial direction. Akgöz and Civalek [73] developed shear deformable beam models including first-order with new shear correction factor and trigonometric shear deformation theories on the basis of the strain gradient elasticity theory. The numerical results showed that the maximum deflections and critical buckling loads obtained by the above two models are in good agreement for all cases.

From the above literature review, it is clearly seen that the current research mainly focuses on the modeling and analysis of microbeam and microplate structures. However, despite their importance, investigations that report on size-dependent shell structures are still very limited. For instance, Zhou and Wang [44] examined the vibration characteristic of simply supported cylindrical microshells conveying fluid and embedded in an elastic medium based on the modified couple stress theory. Zeighampour and Tadi Beni [46] studied vibrations and stability problems of fluid-conveying double-walled carbon nanotubes (DWCNT) based on the Donnell's shell model and the modified couple stress theory. The effect of the van der Waals (vdW) forces was considered between the two walls, and the DWCNT surroundings were