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Effect of micromechanical models on structural responses of functionally graded plates

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ABSTRACT

This paper examines the influence of alternative micromechanical models on the macroscopic behavior of a functionally graded plate based on classical and shear-deformation plate theories. Several micromechanical models are tested to obtain the effective material properties of a two-phase particle composite as a function of the volume fraction of particles which continuously varies through the thickness of a functionally graded plate. The static, buckling, and free- and forced-vibration analyses are conducted for a simply-supported functionally graded plate resting on a Pasternak-type elastic foundation. The volume fraction of particles are assumed to change according to the power-law, Sigmoid, and exponential functions. The governing partial differential equations are solved in the spatial coordinate by Navier solution, while a numerical time integration technique is employed to treat the problem in the time domain. Finally, the numerical results are provided to reveal the effect of explicit micromechanical models such as Voigt, Reuss, Hashin–Shtrikman bounds, and LRVE as well as the semi-explicit model of self-consistent on the static and dynamic displacement and stress fields, critical buckling load, and fundamental frequency.

1. Introduction

While the anisotropic constitution of conventional laminated composites leads to delamination, matrix cracking, and adhesive bond separation due to the stress concentration and geometric discontinuities, functionally graded materials (FGMs) with a spatially continuous transition of material properties alleviate the stress concentration, optimize the structural performance, and simultaneously meet the multiphysical requirements [1,2]. FGMs are composite materials of one or more phases dispersed in a matrix of another phases; they could be associated with particle composites where the volume fraction of particles are tailored in an arbitrary direction. Among the numerous advantages offered by FGMs, one can refer to reduced multiphysical stresses, higher fracture toughness, reduced intensity factor, and improved residual stress distribution [3,4]. Due to the application of functionally graded (FG) thin/thick-walled structures in aerospace, pressure vessels, electronics, and medical industries, the accurate prediction of the behavior of FG structural components is of great significance [5,6].

Shell-like structures made of composite materials and FGMs play a significant role in engineering of weight-efficient structures. To effectively describe the structural behavior, different mathematical models for shell theories have been developed [7]. Because of the computational cost of the three-dimensional (3D) elasticity analysis, lots of efforts have been devoted to develop a consistent equivalent single-layer (ESL) model for structural analysis in which the 3D structural element is replaced by an equivalent two-dimensional (2D) layer with a complex constitutive equation [1]. Since the effect of transverse shear deformation is neglected in the classical laminated plate theory (CLPT), different shear-deformation theories such as the first-order shear deformation theory (FSDT) and third-order shear deformation theory (TSDT) have been introduced. While FSDT assumes a constant shear stress through the thickness of structure and therefore needs a shear correction factor, the TSDT possesses a quadratic variation for shear stresses and thus no shear correction factor is needed [8,9].

Since the emergence of FGMs, the structural behavior of unconstrained/constrained FG components including static, stability, and free- and forced-vibration analyses under multiphysics loading has been the subject of several theoretical and experimental

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investigations [10–20]. For instance, elasticity solutions were obtained in [21–23] for FG beams and plates subjected to electro-mechanical loading. A microstructure-dependent nonlinear beam theory, using the modified, couple stress theory, has recently been reported by Reddy [24] and the corresponding nonlinear static problem of FG beams was studied in [25] using the finite element method. Refined plate theories have also been developed in [26,27] to accurately predict the free-vibration behavior of FG plates. A series of closed-form and semi-analytical solutions for structural responses of FG thick plates under transient thermomechanical loading and a moderately-thick variable stiffness plate were presented by Akbarzadeh et al. [28–31]. The 3D elasticity and finite element models were also given in [32,33] for a dynamic analysis of single/multi-direction FG and sandwich plates. Moreover, to avoid the instability of structures working at different types of multiphysics loading, the buckling analysis of FG components has been conducted in several studies. For instance, closed-form solutions for thermomechanical buckling of FG thin/thick plates have been presented in [34–36].

While most papers in the literature on FGMs employ the simple rule of mixture to obtain the effective material properties, a proper micromechanical model should be used to accurately predict the effective multiphysics properties. As Eshelby elucidated, the objective of micromechanics is to quantify the effect of microstructure on the multiphysics behavior of materials by the application of continuum mechanics to a small-scale [37,38]. Among models in the literature, a few standard micromechanical models could be mentioned. Voigt's [39] and Reuss' [40] approximations are the simplest models used to evaluate the effective material properties of composites. Using the variational principle, Hashin and Shtrikman [41,42] established the upper and lower bounds of the effective material properties. Mori–Tanaka [43] model was introduced to calculate the average internal stresses in the matrix containing an eigenstrain. Benveniste [44] also reformulated the Mori–Tanaka model in order to apply it to composite materials. Finally, the double inclusion methods were proposed by Lielens [45] and Nemmat-Nasser and Hori [46] based on an interpolation of the Mori–Tanaka scheme as a function of the volume fraction of the phases to predict the effective properties of composites.

Several micromechanical models of FGMs have been reviewed in [47–51]. To assess the effect of the micromechanical models on the structural responses of FG plates, this paper presents the static, buckling, and free- and forced-vibration analyses for simply-supported FG plates resting on an elastic foundation. Different micromechanical models are examined to obtain the effective material properties of FGMs with power-law, Sigmoid, and exponential function distributions of volume fraction within the thickness of the plate. Using an analytical method along with a numerical time integration technique, the governing equations are treated and the effects of Voigt, Reuss, Hashin–Shtrikman bounds, LRVE, Tamura, and self-consistent models on the structural responses of the FG plate are investigated.

2. Effective properties of FGMs

FGMs possess a continuous variation of material constituents in spatial coordinates. Such a graded microstructure could be examined as a continuous distribution of discrete particles in a matrix of a reinforced composite. The existing micromechanical models could be extended to predict the effective material properties of FGMs for an entire range of volume fraction (VF) of constituents ($0 \leqslant VF \leqslant 1$) [49].

Consider a two-phase FG plate composed of particles or inclusions and a matrix. While FGMs are typically made from a mixture of ceramics and metals, the material constituent could be, arbitrarily, any two dissimilar materials. The composition of two materials is assumed to vary through the thickness of the plate (z-direction, where z is downward and normal to the middle surface of the plate). The volume fraction of inclusions could vary through the thickness in the form of power-law (P-FGM), Sigmoid (S-FGM), or exponential (E-FGM) [52–55]:

$$VF = VF_t + (VF_b - VF_t)\left(\frac{2z+h}{2h}\right)^n \quad \text{(P-FGM)} \tag{1a}$$

$$VF = \begin{cases} VF_t + (VF_b - VF_t)\left(1 - \frac{1}{2}\left(\frac{h-2z}{h}\right)^n\right) & 0 \leqslant z \leqslant \frac{h}{2} \\ VF_t + (VF_b - VF_t)\left(\frac{1}{2}\left(\frac{h+2z}{h}\right)^n\right) & -\frac{h}{2} \leqslant z \leqslant 0 \end{cases} \quad \text{(S-FGM)} \tag{1b}$$

$$VF = VF_t \exp\left(\ln\left(\frac{VF_b}{VF_t}\right)\left(\frac{2z+h}{2h}\right)^n\right) \quad \text{(E-FGM)} \tag{1c}$$

where VF_t and VF_b are, respectively, the volume fraction of inclusions at the top ($z = -h/2$) and the bottom ($z = h/2$) of FG plates. Furthermore, n and h stand for the non-homogeneity index and thickness of the plate. The non-homogeneity index n could be used to optimize the structural performance of FGMs. In this work, the Voigt, Reuss, Hashin–Shtrikman bounds, LRVE, Tamura, and self-consistent methods are employed to obtain the effective material properties as a function of inclusion volume fraction.

2.1. Voigt and Reuss

The simplest micromechanical model to achieve the equivalent macroscopic material properties is the rule of mixture which was first formulated by Voigt [39]. The Voigt idea was to determine material properties by averaging stresses over all phases with the strain uniformity assumption within the material. The Voigt model, that is frequently used in most FGM analyses, estimates Young's modulus (E) and Poisson's ratio (ν) of FGMs as [56,57]:

$$E(z) = E_i VF(z) + E_m(1 - VF(z)), \\ \nu(z) = \nu_i VF(z) + \nu_m(1 - VF(z)) \tag{2}$$

where the subscripts "i" and "m" denote the material properties of matrix and inclusions (particles). On the other hand, Reuss [39] assumed the stress uniformity through the material and obtained the effective properties as [56,57]:

$$E(z) = \frac{E_i E_m}{E_i(1 - VF(z)) + E_m VF(z)}, \\ \nu(z) = \frac{\nu_i \nu_m}{\nu_i(1 - VF(z)) + \nu_m VF(z)} \tag{3}$$

As shown by Hill [58], the Voigt and Reuss estimations provide, respectively, the upper and lower bounds for Young's modulus for the entire range of inclusion volume fraction. However, as Zimmerman [57] observed, Poisson's ratio could not be bounded to either the Poisson's ratios predicted by Voigt or Reuss, or even the Poisson's ratios of matrix and inclusions. It is worth mentioning that the effective mass density ρ is obtained by the following rule of mixture, regardless of the utilized micromechanical model:

$$\rho(z) = \rho_i VF(z) + \rho_m(1 - VF(z)) \tag{4}$$

2.2. Hashin–Shtrikman bounds

Using the variational principle for heterogeneous linear elasticity, Hashin and Shtrikman derived closed-form expressions for upper and lower bounds of the effective elastic properties. For two-phase materials with a random distribution of spherical particles, the bounds on effective shear (G) and bulk (K) moduli are obtained as [56]: