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Fundamental frequency of a cantilever composite cylindrical shell

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ABSTRACT

Free vibrations of a cantilever composite circular cylindrical shell are considered in this paper. The edge of the shell is fully clamped at one end of the cylinder and is free at the open section of the other end. Variational equations of free vibrations are derived based on Hamilton's principle and the problem is solved using the generalised Galerkin method. Analytical formulas enabling calculations of the fundamental frequency are obtained and verified by comparison with the results of a finite element modal analysis. The efficiency of the analytical solution is demonstrated using numerical examples including the design analysis of composite shells subject to constraints imposed on the fundamental frequency.

1. Introduction

A cylindrical shell is one of the most widely used analytical/mechanical models of thin-walled structures designed in the aerospace and many other industries. A modal analysis of the cylindrical shells is often a necessary part of the design procedures normally employed when the structural components are subjected to loads varying in time. The relevant effective applied theories and methods of analysis were created and developed over the years by many researchers. Results of these studies related to the analysis of dynamic parameters of the cylindrical shells can be found in numerous papers and monographs, e.g. published by Soedel [1], Ventsel and Krauthammer [2], Goldenveizer et al. [3], Volmir [4] and handbooks authored by Gontkevich [5], Leissa [6], Blevins [7], and many others. In most cases considered in the literature, the solutions were found for the thin-walled cylinders having both ends supported in some way. However, there is a certain practical interest to the studies of the dynamic behaviour of the shells with the clamped-free end support (i.e. when the edge of the shell is fully clamped at one end of the cantilever cylinder and is free at the open section of the other end). A brief review of a number of publications considering vibrations of such shells is presented by Leissa [6]. Applications of the Rayleigh–Ritz method to the solution of the vibration problems for cylindrical cantilever shells are reported in the papers by Sharma and Johns [8], Sharma [9,10] and Warburton and Higgs [11]. Tottenham and Shimizu [12] investigated vibrations of cantilever cylindrical shells using a matrix progression method. An integral equation technique was used to determine the natural frequencies of vibration of the clamped-free shells in the article by Srinivasan and Sankaran [13]. Large-amplitude nonlinear vibrations of a cantilever circular cylindrical shell were numerically investigated by Kurylov and Amabili [14]. An asymptotic analysis accounting for edge effect was undertaken by Louhghalam et al. to examine the dynamic characteristics of composite thin cylindrical shells [15]. Soedel [16] used Galerkin's method with shape functions selected in the form of general beam mode shapes to derive a frequency formula for isotropic circular cylindrical shells with various boundary conditions. Effects of boundary conditions on the free vibration characteristics for a multi-layered cylindrical shell using the Ritz method where beam functions were used as the axial modal functions were studied by Lam and Loy [17]. Dai et al. developed a method to study the free vibrations of an isotropic circular shell with various boundary conditions in which the displacements were represented as Fourier series, such that both the governing differential equations based on Flugge's theory and the boundary conditions were satisfied [18]. Semi-analytical approaches to the free vibration analyses of axisymmetric laminated shells with various combinations of boundary conditions were developed by Pinto Correia et al. [19,20] and Santos et al. [21]. Liu et al. derived exact characteristic equations for free vibrations of thin-walled orthotropic cylindrical shells [22].

Most of the results that could be found in the literature related to the vibration analysis of the cantilever orthotropic shells were obtained using numerical solutions, implementation of which requires certain computational effort. In practice, however, it is often sufficient to determine just one, fundamental (i.e. the lowest natural) frequency to assess the stiffness of the structure. The value

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of this frequency provides a convenient criterion of the stiffness and mass efficiency in structural design. This is particularly relevant to aerospace structural components. For example, the criterion can be applied to the design of the cantilever cylindrical shell representing part of the structure of a space telescope. Considering this, it would be advantageous to have an analytical formula that could enable fast and reliable calculations of the fundamental frequency for the clamped-free composite cylindrical shells. This could save a substantial amount of time especially at early stages of design and be particularly useful for the design optimisation. Such a solution has been obtained in this work for the cantilever composite cylindrical shell. Variational equations of free vibrations are derived based on Hamilton's principle and the problem is solved using the generalised Galerkin method which was also successfully applied by the authors to the buckling analysis of a composite cantilever circular cylindrical shell subjected to uniform external lateral pressure [23]. The deflections of moving shell are approximated taking into account an assumption, according to which the shape of the cylinder generator coincides with a fundamental mode shape of a cantilever beam. Cubic and second order algebraic equations enabling the analytical solution of the problem under consideration are derived. Using this solution, a number of example problems have been solved. The results of the analyses are verified by comparison with those obtained using the Finite Element Method (FEM).

2. Problem formulation

Consider a thin-walled composite orthotropic cylindrical shell of radius R and length l with the middle surface referred to the curvilinear coordinate frame $\alpha\beta\gamma$ as shown in Fig. 1. It is assumed that the shell is composed from a large number of elementary plies, so the composite wall material (symmetrical and balanced laminate) can be treated as homogeneous and orthotropic. The edge of the shell at $\alpha = 0$ is fully clamped and the edge at the open end ($\alpha = l$) is free.

The variational equations of free vibrations are derived using Hamilton's principle. In the case under consideration, an action integral has the following form:

$$S = \int_{t_1}^{t_2} L dt \tag{1}$$

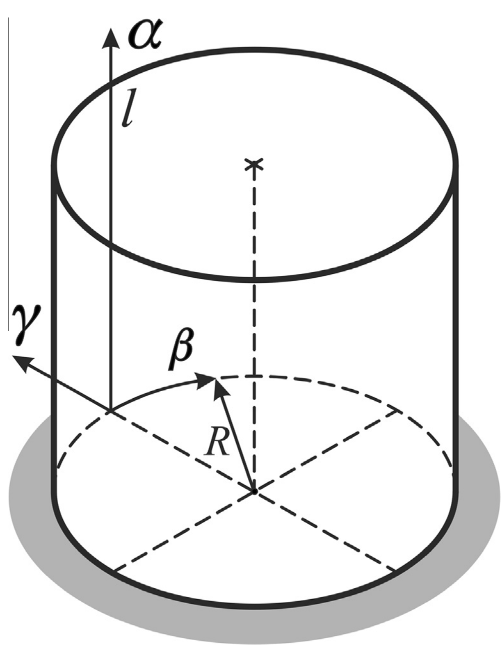

Fig. 1. Clamped-free circular cylindrical shell.

where t is time, $t_2 - t_1$ is the time interval in which the shell motion is considered, and L is the Lagrange function. The latter is equal to the difference between kinetic and potential energies of the shell T and U, i.e.

$$L = T - U \tag{2}$$

In this equation, the kinetic energy of the shell is determined as follows:

$$T = \frac{1}{2}\int_0^l \int_0^{2\pi R} B_\rho \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] d\alpha d\beta \tag{3}$$

where u, v, and w are the displacements of the middle surface of the shell along the axes α, β, and γ (see Fig. 1), respectively; $B_\rho = \rho h$ characterises the inertia properties of the shell (ρ is the material density and h is the shell thickness). The potential energy is presented in the following form [24]:

$$U = \frac{1}{2}\int_0^l \int_0^{2\pi R} (N_\alpha \varepsilon_\alpha + N_{\alpha\beta}\varepsilon_{\alpha\beta} + N_\beta \varepsilon_\beta + M_\alpha \kappa_\alpha + M_{\alpha\beta}\kappa_{\alpha\beta} + M_\beta \kappa_\beta) d\alpha d\beta \tag{4}$$

where N_α, N_β, and $N_{\alpha\beta}$ are the membrane stress resultants; M_α, M_β, and $M_{\alpha\beta}$ are the bending and twisting couples; ε_α, ε_β, and $\varepsilon_{\alpha\beta}$ are the membrane strains and κ_α, κ_β, and $\kappa_{\alpha\beta}$ are the curvatures of the middle surface.

Considering the free vibrations, the displacements, strains and stress resultants and couples can be presented as follows:

$$\begin{Bmatrix} u(\alpha,\beta,t) \\ v(\alpha,\beta,t) \\ w(\alpha,\beta,t) \end{Bmatrix} = \begin{Bmatrix} u(\alpha,\beta) \\ v(\alpha,\beta) \\ w(\alpha,\beta) \end{Bmatrix} \sin\omega t \tag{5}$$

$$\begin{Bmatrix} \varepsilon_\alpha(\alpha,\beta,t) \\ \varepsilon_\beta(\alpha,\beta,t) \\ \varepsilon_{\alpha\beta}(\alpha,\beta,t) \end{Bmatrix} = \begin{Bmatrix} \varepsilon_\alpha(\alpha,\beta) \\ \varepsilon_\beta(\alpha,\beta) \\ \varepsilon_{\alpha\beta}(\alpha,\beta) \end{Bmatrix} \sin\omega t$$

$$\begin{Bmatrix} N_\alpha(\alpha,\beta,t) \\ N_\beta(\alpha,\beta,t) \\ N_{\alpha\beta}(\alpha,\beta,t) \end{Bmatrix} = \begin{Bmatrix} N_\alpha(\alpha,\beta) \\ N_\beta(\alpha,\beta) \\ N_{\alpha\beta}(\alpha,\beta) \end{Bmatrix} \sin\omega t \tag{6}$$

$$\begin{Bmatrix} M_\alpha(\alpha,\beta,t) \\ M_\beta(\alpha,\beta,t) \\ M_{\alpha\beta}(\alpha,\beta,t) \end{Bmatrix} = \begin{Bmatrix} M_\alpha(\alpha,\beta) \\ M_\beta(\alpha,\beta) \\ M_{\alpha\beta}(\alpha,\beta) \end{Bmatrix} \sin\omega t$$

where ω is the circular natural frequency of the shell vibrations. Substituting Eq. (5) into Eq. (3) and Eq. (6) into Eq. (4) yields

$$T = T_{max} cos^2 \omega t, \quad U = U_{max} sin^2 \omega t \tag{7}$$

where T_{max} and U_{max} (the maximum values of the kinetic and potential energies) are

$$\begin{aligned} T_{max} &= \frac{1}{2}\omega^2 \int_0^l \int_0^{2\pi R} B_\rho (u^2 + v^2 + w^2) d\alpha d\beta \\ U_{max} &= \frac{1}{2}\int_0^l \int_0^{2\pi R} (N_\alpha \varepsilon_\alpha + N_{\alpha\beta}\varepsilon_{\alpha\beta} + N_\beta \varepsilon_\beta + M_\alpha k_\alpha + M_{\alpha\beta} k_{\alpha\beta} + M_\beta k_\beta) d\alpha d\beta \end{aligned} \tag{8}$$

Substituting Eq. (7) into Eq. (2), the latter is transformed into the following form:

$$L = T_{max} cos^2 \omega t - U_{max} sin^2 \omega t \tag{9}$$

It is assumed that the time interval in which the shell motion is considered is equal to the period of vibrations with the frequency ω. Then