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Free vibration analysis of the functionally graded sandwich beams by a meshfree boundary-domain integral equation method

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ABSTRACT

Free vibration of the functionally graded (FG) sandwich beams are studied by a meshfree boundary-domain integral equation method. Two sandwich beams, namely, FG core with homogeneous face sheets and homogeneous core with FG face sheets are considered in the study. Based on the two-dimensional elasticity theory, the boundary-domain integral equations are derived by applying the elastostatic fundamental solutions. Radial integration method is employed to transform the domain integrals related to material nonhomogeneity and inertia effect into boundary integrals, hence a meshfree scheme involving boundary integrals only is achieved. Each layer in the sandwich beam is modeled separately and the equilibrium and compatibility conditions are enforced at the interfaces to derive the equations for the whole sandwich beam. Extensive parametric study is presented to examine the influence of material composition, material gradient, layer thickness proportion, thickness to length ratio and boundary conditions on the free vibration of FG sandwich beams.

1. Introduction

Sandwich construction has been used extensively in a variety of engineering fields including aerospace, nuclear, civil and mechanical engineering for its high bending stiffness with low specific weight. Availability of a wide selection of face sheet and core materials makes it possible to obtain multifunctional benefits. For example, use of a viscoelastic layer to a structural sandwich to control vibration and noise propagation. Double-sided coatings for cutting tools help reducing friction and hence extending the edge life for the high-speed, dry and micro machining. Sandwich construction can produce structural elements with high strength and stiffness-to-weight ratio, low weight, good corrosion resistance; good fatigue resistance and functional benefit. These attractive properties have led to a great amount of research efforts in studying sandwich structures. Analysis of sandwich structures is well documented in the books of Plantema [1] and Allen [2].

However, sandwich structures have not been fully exploited in structural applications due to the damage tolerance concerns. Such as core/face sheet delamination, face sheet damage and shear cracking of the core, as a consequence of the thermal expansion mismatch and stress concentration resulted from stiffness discontinuity between layers. It represents a critical design concern since it can susceptible induce failures. As the improvements to the abovementioned deleterious effect, functionally graded materials (FGMs) are believed to alleviate this weak point. The reason is that FGMs are characterized by smooth and continuous variation in the properties from one surface to another, and also can achieve the desirable functionalities. Although a lot of new technologies of researching the FG sandwich structures are at their infancy, their potential advantages appear great.

Venkataraman and Sankar [3] analyzed the elasticity solution for stresses in a sandwich beam with functionally graded core, in which materials were assumed with an exponential variation of the elastic stiffness coefficients across the thickness. The results demonstrated the significant reduction in shear stresses at the face-sheet-core interface achieved by functionally grading the core properties. Deflections and stresses of simply supported FGM sandwich plates under sinusoidal loading were investigated by Zenkour [4] according to different plate theories. A power-law distribution in terms of the volume fraction of the constituents was used to simulate the FG face sheet. He concluded that the gradients in material properties play an important role in determining the response of the FGM plates. Neves et al. [5] also did the static analysis of the power-law FG sandwich plates according to a hyperbolic theory considering Zig-Zag and warping effects. The results leaded to the conclusion that the deflection of the simply supported sand-

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wich plate with FG core or FG skins increases as the power-law exponent of the material increases. An exact thermoelasticity solution for a two-dimensional thick FGM sandwich structures were presented by Shodja et al. [6], they also indicated that a graded core could effectively reduce the face sheet/core interfacial shear stress. Apetre et al. [7] addressed the low-speed impact response of sandwich beams with FG core. The results revealed that FG cores could be used effectively to mitigate or completely prevent impact damage in sandwich composites. Model sandwich structures comprising of graded core with bilinear variation of volume fraction of hollow microballoons were considered for experimental and numerical simulations by Kirugulige et al. [8]. They showed that a decrease of stress intensity factors was obtained by using a FGM core. Etemadi et al. [9] conducted three-dimensional finite element simulations for analyzing low velocity impact behavior of sandwich beams with a FG core, the results revealed that the maximum contact force increased and the maximum strain decreased compared to those of sandwich beams with a homogeneous core. Buckling analysis of the power-law sandwich plates with FG skins using a new quasi-3D hyperbolic sine shear deformation theory and collocation with radial basis functions was presented by Neves et al. [10]. They concluded that the critical buckling loads decrease as the power-law exponent increases. As the core to plate total thickness ratio increases, the buckling loads increases as well.

Vibration analysis always plays an important role to exploit the mechanics world of the novel constructions. Chehel Amirani et al. [11] analyzed the free vibration of sandwich beam with FG core by using the element free Galerkin method. The Mori–Tanaka method was used as the mircomechanics technique to determine the effective properties of FG core. Zenkour [12] studied the free vibration of the simply supported FG sandwich plate by the sinusoidal shear deformation plate theory. Li et al. [13] presented three-dimensional vibration analytical solutions for multi-layer FGM sandwich plates based on Ritz method in conjunction with Chebhyshev polynomial series. Neves et al. conducted the free vibration of the FG plates and FG sandwich plates by a thickness-stretching sinusoidal shear deformation theory [14], a new hyperbolic sine shear deformation theory [15], and a thickness-stretching higher-order shear deformation theory [16]. All of them assumed that the properties of the FGMs follow the power law volume fraction function. Furthermore, the three-dimensional models may be computationally involved and expensive. Hence, there is a growing appreciation of the importance of applying two-dimensional theories for the evolution of the accurate structural analysis. More effective numerical solution scheme is also necessary for predicting the vibration behaviors of FG sandwich structures, especially for the free vibration behaviors of the exponentially FG sandwich beams which are rarely studied in the literature.

In this paper, a meshfree boundary-domain integral equation is presented for studying free vibration of sandwich beams composed of FGM. Based on the two-dimensional elasticity theory, the boundary-domain integral equations for each layer of the FG sandwich beams are derived initially by using the elastostatic fundamental solutions. The resulting domain integrals are due to the material nonhomogeneity and the inertial effect. By applying the radial integration method, these domain integrals are transformed into boundary integrals which can be evaluated by a meshfree scheme. Then, enforcing the equilibrium and compatibility conditions at the interfaces in the sandwich beam, an eigenvalue system involving the system matrices with only boundary integrals for free vibration of the sandwich beams are obtained. Numerical study is carried out to evaluate the effect of various important parameters on the free vibration of the FG sandwich beams.

2. FG sandwich beam

In this study, two types of the FG sandwich beams are studied, namely, a non-symmetrical FG core with homogeneous face sheets and a homogeneous core with FG face sheets as depicted in Fig. 1. By assuming that the layers are perfectly bonded to each other. The total length and height of these sandwich beams are denoted by L and h_t. In the first type of sandwich beam, the top and bottom homogeneous face sheets are pure ceramic and pure steel, respectively, while the FG core consists of material with continuous variation of steel from the bottom face to ceramic on the top face. For the other type of beam, the core is ceramic with face sheet properties varying inwardly from steel to ceramic. The material parameters of the ceramic and the steel are described in Table 1. For the FG layer, the Young's modulus and the mass density are varying gradually in the transverse direction according to an exponential function described in Eqs. (1) and (2), while the Poisson's ratio is a constant. This functional form of property variation has been recognized convenient in solving elasticity problems by many researchers [17–19].

$$E(x_2) = E_b e^{\beta x_2}, \quad \text{where} \quad \beta = \frac{1}{T_m} \ln\left(\frac{E_t}{E_b}\right), \tag{1}$$

$$\rho(x_2) = \rho_b e^{\gamma x_2}, \quad \text{where} \quad \gamma = \frac{1}{T_m} \ln\left(\frac{\rho_t}{\rho_b}\right), \tag{2}$$

where E_t, ρ_t are the Young's modulus and mass density for the top face constituent of the FG layer, and E_b, ρ_b are for the bottom face constituent. β and γ represent the FG gradation parameters for the Young's modulus and mass density respectively. x_2 denotes the Cartesian coordinates variable in the transverse direction and T_m is the thickness of the FG layer. The through thickness variation of the Young's modulus for these two type of sandwich beams are shown in Fig. 2.

3. Problem formulation

Based on the two-dimensional elasticity theory, the governing differential equations of the undamped steady-state elastodynamics for each single layer of the considered FG sandwich beams can be expressed as [17]

$$\sigma_{ij,j}(\boldsymbol{x}) + \omega^2 \rho(\boldsymbol{x}) u_i(\boldsymbol{x}) = 0, \tag{3}$$

where σ_{ij}, ρ and u_i is the stress tensor, mass density and displacements and ω is the vibration frequency. A comma after a quantity represents spatial derivatives and repeated indexes denote summation.

The elasticity tensor c_{ijkl} for nonhomogeneous isotropic material is described in the form of

$$c_{ijkl}(\boldsymbol{x}) = \mu(\boldsymbol{x}) c^0_{ijkl}, \quad \text{where}$$
$$c^0_{ijkl} = \frac{2\nu}{1-2\nu}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}, \tag{4a, b}$$

where c^0_{ijkl} is the elastic tensor of a "fictitious" reference homogeneous material with $\mu = 1$. The shear modulus $\mu(x)$ is related to $E(x)$ as $\mu(\boldsymbol{x}) = E(\boldsymbol{x})/2(1 + \nu)$. $\mu(\boldsymbol{x})$ varies continuously with the coordinates for the FG layers, while it keeps a constant for the homogeneous layer. δ_{ij} is the Kronecker delta. By taking the elastostatic displacement fundamental solutions $U_{ij}(\boldsymbol{x}, \boldsymbol{y})$ as the weight function, the weak-form of the equilibrium equation (3) can be obtained as

$$\int_\Omega [\sigma_{jk,k} + \rho\omega^2 u_j] \cdot U_{ij} d\Omega = 0. \tag{5}$$

Substitution of the generalized Hooke's law $\sigma_{ij} = c_{ijkl}u_{k,l} = \mu(\boldsymbol{x})c^0_{ijkl}u_{k,l}$ and application of the Gauss's divergence theorem, the boundary-domain integral equations yield as