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The vibration and buckling of sandwich cylindrical shells covered by different coatings subjected to the hydrostatic pressure

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ABSTRACT

The vibration and buckling of sandwich cylindrical shells covered by different types of coatings, such as functionally graded (FG), metal and ceramic coatings and subjected to a uniform hydrostatic pressure using first order shear deformation theory (FOSDT) is discussed. Four types of sandwich cylindrical shells are considered. The volume fraction of FG coatings varies according to a simple power law function of thickness coordinate, while that of the core equals unity. The effective material properties of FG coatings are assumed to be graded in the thickness direction according to an exponential law distribution. The equations of motion of FG sandwich cylindrical shells are deduced using the FOSDT. The closed-form solutions for non-dimensional frequencies and critical hydrostatic pressures are obtained. The influences of compositional profiles of coatings, shear stresses and sandwich shell characteristics on the non-dimensional frequencies and critical hydrostatic pressures for FG and homogeneous sandwich cylindrical shells are discussed.

1. Introduction

Functionally graded materials (FGMs) are multifunctional materials that contain spatial variations in composition and microstructure of the specific purpose of controlling thermal variations, structural or functional properties. These materials are now at the forefront of materials research receiving worldwide attention [1]. In recent years, various methods have been developed to study the stability and vibration problems of FGM shells based on shear deformation theories that assert the importance of FGM shells [2–21].

Typical sandwich structure consists of two thin face sheets embedding thick and soft core. The coatings are used mainly for protection of metal or ceramic substrates against oxidation, heat penetration, abrasion, corrosion and etc. Due to their outstanding flexural stiffness-to-weight ratio compared to other constructions, sandwich structures are widely used in many fields of technology, such as aerospace, aeronautic, marine, vehicles and etc. [22]. The main disadvantage of conventional sandwich structures is the presence of strong discontinuity stresses at the face sheet-core interface which may introduce serious delamination problems. To overcome these disadvantages, designers have created new types of sandwich structures: containing FG core or with FG coatings. FG coatings were produced by various methods including vapor deposition (PVD), chemical vapor deposition (CVD), plasma spraying, sputtering, pulsed laser deposition (PLD), sol–gel techniques, as well as build-up welding [23,24]. Use FG layers can lead to significant mitigating the intensity of interfacial stress. The separation of FG coatings by a homogeneous core increases the bending rigidity of structures at expenses of small weight [25]. Among a great number of studies on the FGM structures, an interesting issue is the bending, buckling and vibration analysis of sandwich structures with FG coatings. Sodya et al. [26] presented a thermo-elasticity analysis for a two-dimensional thick composite consisting of homogeneous and FG layers, which governing equation is reduced to a fourth order inhomogeneous partial differential equation and solved exactly using Fourier series method. Shen and Li [27] presented compressive postbuckling under thermal environments and thermal postbuckling due to heat conduction for a simply supported sandwich plate with FGM face sheets using a two-step perturbation technique. Li et al. [28] studied Free vibration of sandwich rectangular plates with FGM face sheet and homogeneous core and the sandwich with homogeneous face sheet and FGM core, based on the three-dimensional linear theory of elasticity. Zenkour and Sobhy [29] studied the thermal buckling of FG coated sandwich plates using the SDT containing the higher- and first-order shear deformation theories and CST, as special cases. Tornabene et al. [30] studied the dynamic behavior of functionally graded materials (FGMs) and laminated doubly curved shells and panels of revolution with a free-form meridian, using first-order shear deformation

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theory (FSDT) and the generalized DQM. Kiani and Eslami [31] and Kiani et al. [32] investigated thermal and mechanical buckling and post-buckling behaviors of perfect and imperfect sandwich plates with FGM face sheets under uniform temperature rise loading, employing the single mode approach combined with Galerkin technique. Neves et al. [33] used quasi-3d higher-order shear deformation theory (HOSDT) and meshless technique to the static, free vibration and buckling analysis of isotropic and sandwich FG plates. Zaki et al. [34] presented stresses and strains analyses of clamped plates with FG coatings, using two-dimensional elasticity theory and superposition method to find a series solution. Dozio [35] presented the formulation of advanced two-dimensional Ritz-based models for accurate prediction of natural frequencies of thin and thick sandwich plates with core made of FGM, using an entire family of higher-order layerwise and equivalent single-layer theories. Sobhy [36] studied the buckling and free vibration of plates with exponentially graded (EG) coatings surrounding by elastic medium under various boundary conditions, using the sinusoidal shear deformation theory. Fazzolari and Carrera [37] studied free vibration analysis of doubly curved FGM shells and sandwich shells with FGM core use the Ritz minimum energy method, based on the use of the principle of virtual displacements (PVD), is combined with refined equivalent single layer (ESL) and zig-zag (ZZ) shell models hierarchically generated by exploiting the use of Carrera's Unified Formulation (CUF), in order to engender the hierarchical trigonometric Ritz formulation (HTRF). Malekzadeh and Ghaedsharaf [38] studied the three-dimensional (3D) free vibration of laminated cylindrical panels with finite length and FG core and coating employing layerwise-differential quadrature method (LW-DQM). Hamidi et al. [39] developed a new four variable refined plate theory for bending response of functionally graded sandwich plates under thermo mechanical loading.

In addition, some works have produced for the buckling of cylindrical and conical shells with FG coatings under different static loading conditions using CST. Sofiyev and Kuruoglu [40] studied torsional buckling and free vibration of the cylindrical shell with FG coatings surrounded by an elastic medium using the CST. Deniz [41] studied the non-linear static stability analysis of truncated conical shell with functionally graded composite coatings in the finite deflection, using the CST.

Nevertheless, stability and vibration analysis of sandwich shells with FG coatings based on the SDT has not been studied sufficiently. The purpose of this study is the solution of such problems.

2. Formulation of the problem

Consider a sandwich cylindrical subjected to a uniform hydrostatic pressure, P, as shown in Fig. 1. The origin of the coordinate system is taken at the left end of the reference surface of a sandwich cylindrical shell, where x and y are the longitudinal and circumferential direction, and ζ axis normal to them. The length, radius and total thickness of the sandwich cylindrical shell are L, R and h, respectively.

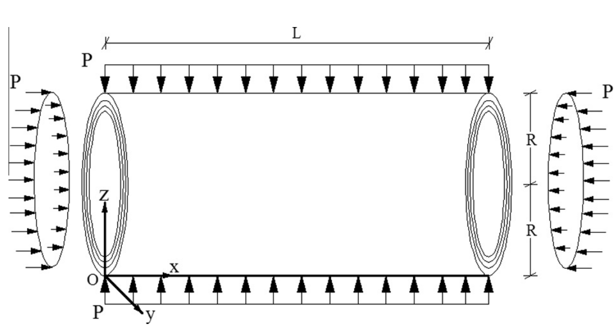

Fig. 1. Nomenclature and coordinate system of sandwich cylindrical shell.

The inner and outer surfaces of the sandwich cylindrical shell are covered with coatings consisting of different materials. The plane geometry of the sandwich cylindrical shells is shown in Fig. 2. The thickness of each coating is h_{coat}, while the thickness of the metal and ceramic core are h_m and h_c, respectively. Four types of sandwich cylindrical shells, namely, (a) the sandwich cylindrical shell with FG1 coatings and ceramic-rich core (FG1–C–FG1), (b) the sandwich cylindrical shell with FG2 coatings and metal-rich core (FG2–M–FG2), (c) the sandwich cylindrical shell with metal-rich coatings and ceramic-rich core (M–C–M), (d) the sandwich cylindrical shell with ceramic-rich coatings and metal-rich core (C–M–C) are considered (see, Fig. 2). These four types of sandwich shells covered by FG or homogenous isotropic coatings will be discussed in this study.

The material properties of FG1 and FG2 coatings are varying smoothly in the ζ direction only. We assume that the composition is varied from the interfaces to the outer and inner surfaces, i.e. the outer ($\zeta = h_0 = -h/2$) and inner ($\zeta = h_3 = +h/2$) surfaces of the cylindrical shell are core, whereas the interfaces (h_1, h_2) are metal-rich or ceramic-rich. The volume fraction of the FG1 and FG2 coatings varies according to a simple power law function of ζ while that of the core equals unity, and they are given as:

$$\begin{aligned} V^{(1)} &= \left(\frac{2\overline{\zeta}+1}{2\zeta_2+1}\right)^N, \quad \zeta_0 \leqslant \overline{\zeta} < \zeta_1, \; (\zeta_0 = -0.5) \\ V^{(2)} &= 1, \quad \zeta_1 \leqslant \overline{\zeta} \leqslant \zeta_2 \\ V^{(3)} &= \left(\frac{2\overline{\zeta}-1}{2\zeta_3-1}\right)^N, \quad \zeta_2 < \overline{\zeta} \leqslant \zeta_3 \; (\zeta_3 = 0.5) \end{aligned} \tag{1}$$

where $\overline{\zeta} = \zeta/h;\ \zeta_{i-1} = h_{i-1}/h\ (i = 1, 2, \ldots, 4)$ and N is the heterogeneity parameter which takes values greater than or equal to zero. It is noted that the core is independent of the value of N which is homogeneous isotropic (metal or ceramic). The value of N equaling to zero represents a homogeneous isotropic (metal-rich or ceramic-rich) shell and the value of it equaling to infinity represent metal–ceramic–metal (M–C–M) or ceramic–metal–ceramic (C–M–C) sandwich cylindrical shells. The above power law assumption reflects a simple rule of mixtures used to obtain the effective material properties of FG1 and FG2 coatings.

The effective material properties, like Young's modulus, Poisson's ratio and mass density of outer FG1 and FG2 coatings by the rule of mixture [41]

$$E^{(1)}_{FG1}(\overline{\zeta}) = E_m e^{V^{(1)}\ln(E_c/E_m)}, \quad \nu^{(1)}_{FG1}(\overline{\zeta}) = \nu_m e^{V^{(1)}\ln(\nu_c/\nu_m)}, \quad \rho^{(1)}_{FG1}(\overline{\zeta}) = \rho_m e^{V^{(1)}\ln(\nu_c/\nu_m)} \tag{2a}$$

$$E^{(1)}_{FG2}(\overline{\zeta}) = E_c e^{V^{(1)}\ln(E_m/E_c)}, \quad \nu^{(1)}_{FG2}(\overline{\zeta}) = \nu_c e^{V^{(1)}\ln(\nu_m/\nu_c)}, \quad \rho^{(1)}_{FG2}(\overline{\zeta}) = \rho_c e^{V^{(1)}\ln(\nu_m/\nu_c)} \tag{2b}$$

and the inner FG1 and FG2 coatings can be expressed as

$$E^{(3)}_{FG1}(\overline{\zeta}) = E_m e^{V^{(3)}\ln(E_c/E_m)}, \quad \nu^{(3)}_{FG1}(\overline{\zeta}) = \nu_m e^{V^{(3)}\ln(\nu_c/\nu_m)}, \quad \rho^{(3)}_{FG1}(\overline{\zeta}) = \rho_m e^{V^{(3)}\ln(\nu_c/\nu_m)} \tag{3a}$$

$$E^{(3)}_{FG2}(\overline{\zeta}) = E_c e^{V^{(3)}\ln(E_m/E_c)}, \quad \nu^{(3)}_{FG2}(\overline{\zeta}) = \nu_c e^{V^{(3)}\ln(\nu_m/\nu_c)}, \quad \rho^{(3)}_{FG2}(\overline{\zeta}) = \rho_c e^{V^{(3)}\ln(\nu_m/\nu_c)} \tag{3b}$$

where E_m, ν_m, ρ_m and E_c, ν_c, ρ_c are the Young's modulus, Poisson's ratio and density of the metal and ceramic surfaces of FG1 or FG2 coatings.

The variation of Young's modulus, Poisson's ratio and density of sandwich cylindrical shell covered by FGi (i = 1,2) coatings are given as [36]:

$$\left[E(\overline{\zeta}), \nu(\overline{\zeta}), \rho(\overline{\zeta})\right] = \begin{cases} E^{(1)}_{FGi}, \nu^{(1)}_{FGi}, \rho^{(1)}_{FGi} & -0.5 \leqslant \overline{\zeta} < \zeta_1 \\ E^{(2)}_i, \nu^{(2)}_i, \rho^{(2)}_i & \zeta_1 \leqslant \overline{\zeta} \leqslant \zeta_2 \\ E^{(3)}_{FGi}, \nu^{(3)}_{FGi}, \rho^{(3)}_{FGi} & \zeta_2 < \overline{\zeta} \leqslant 0.5 \end{cases}, \quad (i = 1, 2) \tag{4}$$

where $E^{(2)}_1 = E^{(2)}_c, \nu^{(2)}_1 = \nu^{(2)}_c, \rho^{(2)}_1 = \rho^{(2)}_c$ and $E^{(2)}_2 = E^{(2)}_m, \nu^{(2)}_2 = \nu^{(2)}_m, \rho^{(2)}_2 = \rho^{(2)}_m$.