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Free vibration analysis of rotating functionally graded carbon nanotube-reinforced composite truncated conical shells

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ABSTRACT

The influences of centrifugal and Coriolis forces on the free vibration behavior of rotating carbon nanotube reinforced composite (CNTRC) truncated conical shells are examined. The material properties of functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) are assumed to be graded in the thickness direction and are estimated through a micromechanical model. The governing equations are derived based on the first-order shear deformation theory (FSDT) of shells using Hamilton's principle. The initial mechanical stresses are obtained by solving the dynamic equilibrium equations. The differential quadrature method (DQM) is adopted to discretize the equations of motion and the related boundary conditions. After demonstrating the convergence and accuracy of the presented approach, the effects of angular velocity, Coriolis acceleration, geometrical parameters, type of distribution and volume fractions of carbon nanotubes on the frequency parameters of the CNTRC truncated conical shells are studied. The results reveal that the influences of the type of carbon nanotube distribution and its volume fraction on the frequency parameters depend on the semi vertex angle and angular velocity of the shells and the frequency parameters of the shell with FG asymmetric carbon nanotube distribution can become greater than those of the case with FG symmetric distribution ones.

1. Introduction

Truncated composite conical shells are being used as structural components in modern industries of different engineering fields such as aerospace, mechanical and nuclear engineering. In some applications, these structural elements undergo rotations with constant angular velocities. The induced initial mechanical stresses by the centrifugal forces together with Coriolis forces can change their vibration characteristics to be become different from those of stationary shells. Hence, accurate evaluation of their vibration characteristics under these working conditions becomes essential for their engineering design and manufacture.

Usually, the conventional composites require a high content (greater than 10% by weight) of the inorganic fillers to impart the desired mechanical properties. On the other hand, it has been found that carbon nanotubes (CNTs) have extraordinary mechanical and electrical properties over the carbon fibers [1]. These facts encouraged the researchers to replace the conventional micro-sized fibers with carbon nanotubes (CNTs) to develop the low density, high strength and elastic modulus carbon nanotube-reinforced polymer composites [2–9]. Since these advanced composite material structures contain a low percentage of CTNs (2–5% by weight) [7–9], in order to effectively use of the CNTs, Shen [10] suggested considering graded distribution of CNTs in the matrix. He showed [10] that the nonlinear bending behavior of CNTRC plates can be considerably improved through the use of a functionally graded (FG) distribution of CNTs in the matrix. Since then and by applying the concept of functionally graded materials (FGMs) to the nanocomposites (FG-CNTRCs), some researchers studied the global behavior of structural elements made of FG-CNTRCs; see for example Refs. [10–12 a recent work of beam, plate and shell].

The free vibration analyses of rotating isotropic and laminated composite truncated conical shells have been studied extensively in previous works; see for example Refs. [13–16]. In addition, the free vibration behaviors of stationary and rotating FG truncated conical shells have been investigated in the recent years [17–24]. However, to the best of authors' knowledge, the only available works related to the free vibration analysis of carbon nanotube-reinforced composite shells are limited to those of stationary cylindrical shells [12,25,26].

To the best of authors' knowledge, the influence of the initial mechanical stresses due to rotation on the free vibration behavior of the FG-CNTRC truncated conical shells has not been investigated

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yet. Due to the practical importance of this problem and also lack of information in the open literature in this regards, this problem is considered in this paper. For this purpose, the dynamic equilibrium equations and the equations of motion for the free vibration of the shell around the dynamic equilibrium state are extracted using Hamilton's principle. The differential quadrature method (DQM) as an efficient and accurate numerical tool [13,14,19,20,23,24,27] is employed to discretize the governing differential equations subjected to the related boundary conditions. After validating the presented formulation and method of solution, the effects of different geometrical parameters, material properties and boundary conditions on the frequency parameters of rotating FG-CNTRC truncated conical shells are studied.

2. Governing equations

2.1. Geometry and material properties definitions

Consider a truncated conical shell made of CNTRCs in which the distribution of CNTs is graded along the thickness direction (FG-CNTRCs). The largest mean radius of the shell is denoted as R_2, smallest mean radius R_1, thickness h, semi-vertex angle β and length L, which rotates about its axis with a constant angular velocity Ω as shown in Fig. 1. The conical coordinate system with coordinate variables (s, θ, z), which are shown in Fig. 1, is used to label the material points of the CNTRCs shell in the undeformed reference configuration. In addition, in Fig. 1(c)–(f), four different profiles of CNTs distribution along the shell thickness considered in this study are shown. In the first case, the CNTs has a uniformly distribution through the shell thickness, which is referred to UD type. In the second case, the CNTs have a graded distribution with mid-plane symmetry and both inner and outer surfaces are CNTs-rich (FG-X type). In the third type, the inner surface is CNTs-rich, whereas the outer surface is matrix-rich (FG-$\boldsymbol{\Lambda}$ type). In the last case, the distribution of CNTs is such that the inner surface is matrix-rich and the outer surface is CNTs-rich (FG-V type).

In order to model the influences of the carbon nanotubes on the overall properties of the composite shell, the extended rule of mixture as a simple and convenient micromechanics model [25] is used. According to it, the effective Young's modulus and shear modulus of CNTRCs can be expressed as [25]:

$$E_{11}=\eta_1 V_{CN}E_{11}^{CN}+V_M E^M,\quad \frac{\eta_2}{E_{22}}=\frac{V_{CN}}{E_{22}^{CN}}+\frac{V_M}{E^M}\quad ,\frac{\eta_3}{G_{12}}=\frac{V_{CN}}{G_{12}^{CN}}+\frac{V_M}{G^M} \tag{1a-c}$$

where E_{11}^{CN} , E_{22}^{CN} and G_{12}^{CN} are the Young's and shear moduli of the CNTs, E^M and G^M are the corresponding properties for the matrix, and the η_j (j = 1, 2, 3) are the CNT efficiency parameters, respectively. In addition, V_{CN} and V_M are the volume fractions of the CNT and the matrix, which satisfy the relationship of $V_{CN} + V_M = 1$.

The material properties of the FG-CNTRC shell vary continuously and smoothly in the thickness direction of the shell, i.e. z-direction. Similarity, in order to examine the effect of different CNT distributions on the free vibration characteristics of a FG-CNTRC truncated conical shell, various types of material profiles through the shell thickness are considered. In this work, we assume only linear distribution of the CNT volume fraction for the different types of the CNTRC truncated conical shell that can readily be achieved in practice, as follows:

$$\text{FG-V}:\quad V_{CN}=\left(\frac{2z+h}{h}\right)V_{CN}^{*} \tag{2}$$

$$\text{FG}-\Lambda:\quad V_{CN}=-\left(\frac{2z-h}{h}\right)V_{CN}^{*} \tag{3}$$

$$\text{FG-X}:\quad V_{CN}=4\left(\frac{|z|}{h}\right)V_{CN}^{*} \tag{4}$$

in which,

$$V_{CN}^{*}=\frac{w_{CN}}{w_{CN}+\left(\frac{\rho^{CN}}{\rho^{M}}\right)-\left(\frac{\rho^{CN}}{\rho^{M}}\right)w_{CN}} \tag{5}$$

where w_{CN} is the mass fraction of nanotube, and ρ^{CN} and ρ^M are the densities of carbon nanotube and matrix, respectively. Note that $V_{CN} = V_{CN}^*$ corresponds to the uniformly distributed CNTRC (UD-CNTRC) truncated conical shell, referred to as UD. It assumed that in all cases the CNTRC shells have the same CNTs mass fraction.

Similarly, Poisson's ratio ν and mass density ρ can be calculated by

$$\nu_{12}=V_{CN}\nu_{12}^{CN}+V_M\nu^M,\quad \rho=V_{CN}\rho^{CN}+V_M\rho^M \tag{6a, b}$$

where ν_{12}^{CN} and ν^M are Poisson's ratios of carbon nanotube and matrix, respectively.

2.2. Initial dynamic equilibrium configuration

Since the free vibration of the CNTRC shell takes place around its dynamic equilibrium state due to its steady state rotation, this configuration and the related initial deformation and stresses should be evaluated. Because of the axisymmetric geometry and loading of the shell, the only nonzero displacement components at an arbitrary material point of the shell are the axial and transverse components $(\bar{u}_0,\bar{w}_0)$, respectively. Based on the first-order shear deformable theory of shells, these nonzero displacement components can be approximated in the thickness direction as:

$$\bar{u}_0(s,z)=u_0(s)+z\varphi_0^s(s),\quad \bar{w}_0(s,z)=w_0(s) \tag{7a, b}$$

where u_0 and w_0 are the displacement components of a material point on the mid-surface of the shell along the s and $z-$ directions, respectively; also φ_0^s is the bending rotation of the unit normal to the mid-surface of the shell about the θ-axis. Hereafter, a subscript '0' is used to represent the deformation field variables and also the stress components in the initial dynamic equilibrium state.

Using the principle of virtual work, the dynamic equilibrium equations and the related boundary conditions can be derived in terms of the displacement and the rotation components as,

Equilibrium equations:

$$\begin{aligned}\delta u_0:\quad & A_n\frac{d^2u_0}{ds^2}+\left(\frac{A_n\sin\beta}{r}\right)\frac{du_0}{ds}-\left(\frac{A_m\sin^2\beta}{r^2}\right)u_0\\&+\left(\frac{A_{mn}\cos\beta}{r}\right)\frac{dw_0}{ds}-\left(\frac{A_m\sin2\beta}{2r^2}\right)w_0\\&+\left(\frac{B_n\sin\beta}{r}\right)\frac{d\varphi_0^s}{ds}-\left(\frac{B_m\sin^2\beta}{r^2}\right)\varphi_0^s=0\end{aligned} \tag{8}$$

$$\begin{aligned}\delta w_0:\quad & \left(\frac{A_{mn}\cos\beta}{r}\right)\frac{du_0}{ds}+\left(\frac{A_m\sin2\beta}{2r^2}\right)u_0-A_r\frac{d^2w_0}{ds^2}\\&-\left(\frac{A_r\sin\beta}{r}\right)\frac{dw_0}{ds}+\left(\frac{A_m\cos^2\beta}{r^2}\right)w_0+\left(\frac{B_{mn}\cos\beta}{r}-A_r\right)\frac{d\varphi_0^s}{ds}\\&+\left(\frac{B_m\sin2\beta}{2r^2}-\frac{A_r\sin\beta}{r}\right)\varphi_0^s=I_{11}\Omega^2(r+w_0)\end{aligned} \tag{9}$$

$$\begin{aligned}\delta\varphi_0^s:\quad & B_n\frac{d^2u_0}{ds^2}+\left(\frac{B_n\sin\beta}{r}\right)\frac{du_0}{ds}-\left(\frac{B_n\sin^2\beta}{r^2}\right)u_0\\&+\left(\frac{B_{mn}\cos\beta}{r}-A_r\right)\frac{dw_0}{ds}-\left(\frac{B_m\sin2\beta}{2r^2}\right)w_0\\&+D_n\frac{d^2\varphi_0^s}{ds^2}+\left(\frac{D_n\sin\beta}{r}\right)\frac{d\varphi_0^s}{ds}-\left(A_r+\frac{D_m\sin^2\beta}{r^2}\right)\varphi_0^s=0\end{aligned} \tag{10}$$

Boundary conditions at the ends $s = 0$ and $s = L$: