Composite Structures 117 (2014) 298–308

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Accurate finite-element modeling of wave propagation in composite and functionally graded materials

A. Idesman *

Department of Mechanical Engineering, Texas Tech University, Lubbock, TX 79409-1021, USA

ARTICLE INFO

Article history:
Available online 8 July 2014

Keywords:
Waves
Composite and functionally graded materials
Time integration
Finite elements
Spurious oscillations

ABSTRACT

For the first time, we have obtained accurate numerical solutions for wave propagation in inhomogeneous materials under impact loading. We have extended the earlier developed numerical approach for elastodynamics problems in homogeneous materials to inhomogeneous materials. The approach includes the two-stage time-integration technique with the quantification and the filtering of spurious oscillations, the special design of non-uniform meshes as well as includes the standard finite elements and the elements with reduced dispersion. Similar to wave propagation in homogeneous materials in the 1-D case, we have obtained very accurate results for composite and functionally graded materials using the linear elements with lumped mass matrix and the explicit central difference method. We have also shown that specific non-uniform meshes yield much more accurate results compared to uniform meshes. We have also shown the efficiency of the finite elements with reduced dispersion compared with the standard finite elements.

1. Introduction

In the paper we will consider the accurate finite element modeling of wave propagation problems in inhomogeneous materials such as composite and functionally graded materials. The known finite element techniques for inhomogeneous materials (e.g., see [1–12]) are usually based on the explicit introduction of material properties as functions of the coordinates in the expressions for the mass and stiffness matrices. In some approaches, these properties are assumed to be constant within each element (but different for different elements), in other approaches they are approximated in terms of the standard finite element shape functions (the so-called graded elements). As shown in [4], the difference in numerical results between these formulations is not big. However, we have not seen in the literature the recommendations of how to select the dimensions of the finite elements depending on the variation of material properties. In many publications (e.g., see [1–12]), uniform meshes and meshes independent of material properties are used in calculations. However, the properties for composite and functionally graded materials can significantly differ in different locations, and an improper selection of the sizes of finite elements may lead to very inaccurate results or to a prohibitively large computation time for wave propagation problems if very fine meshes are used. Another issue with the finite element modeling of wave propagation in inhomogeneous material is related to the appearance and the quantification of spurious high-frequency oscillations. These oscillations can be very large (especially under impact loading) and can destroy the accuracy of numerical results. We should mention that accurate numerical results for wave propagation in inhomogeneous materials are very important and necessary for many engineering applications (e.g., see [13] for shock mitigation by the use of functionally graded materials).

In our previous papers [14–21] we have developed the new numerical approach for wave propagation in homogeneous materials. This approach includes the two-stage time-integration technique with the stage of basic calculations as well as the filtering stage with the quantification and the filtering of spurious oscillations. This technique yields accurate numerical results for wave propagation problems at any loading (including impact loading) in the 1-D, 2-D and 3-D cases. The approach is implemented for implicit and explicit time-integration methods as well as for the low- and high-order standard finite elements, the spectral elements, isogeometric elements, and the linear elements with reduced dispersion. In this paper we will extend our numerical approach for elastodynamics problems to inhomogeneous materials. It will include the two-stage time-integration technique with the quantification and filtering of spurious oscillations, the special design of non-uniform meshes as well as will include the standard

* Tel.: +1 806 742 3563; fax: +1 806 742 3540.
E-mail address: alexander.idesman@ttu.edu

http://dx.doi.org/10.1016/j.compstruct.2014.06.032
0263-8223/

finite elements and the elements with reduced dispersion. Similar to wave propagation in homogeneous materials in the 1-D case, we will obtain very accurate results for composite and functionally graded materials using the linear elements with the lumped mass matrix and the explicit central difference method with the time increments equal to the stability limit. We will also show that specific non-uniform meshes yield much more accurate results compared to uniform meshes. For the first time, we will obtain accurate numerical solutions for wave propagation in inhomogeneous materials under impact loading. These numerical solutions do not include spurious oscillations and converge at mesh refinement (as shown below, existing approaches may yield divergent results at mesh refinement; see Figs. 12 and 14(a) and (b)). We will also show the efficiency of the linear finite elements with reduced dispersion compared with the standard linear finite elements. In our previous papers [15,20] we have shown that for elastodynamics problems the linear elements with reduced dispersion are more efficient (i.e., require less computation time at the same accuracy) than the standard quadratic elements (see [15]) and at moderate observation times they are more efficient than the high-order standard, spectral and isogeometric elements (see [20]).

The paper consists of the extension of the two-stage time-integration technique to wave propagation in inhomogeneous materials (Section 2.1), the design of non-uniform meshes for these problems (Section 2.2), and the examples of the accurate modeling of wave propagation problems in composite and functionally graded materials under impact loading (the most challenging case for accurate simulations) using the standard linear finite elements as well as the linear elements with reduced dispersion (Section 3).

2. Numerical technique

The application of the space discretization to a system of partial differential equations for transient acoustics or transient linear elastodynamics leads to a system of ordinary differential equations in time

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{C}\dot{\boldsymbol{U}} + \boldsymbol{K}\boldsymbol{U} = \boldsymbol{R}, \tag{1}$$

where $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ are the mass, damping, and stiffness matrices, respectively, $\boldsymbol{U}$ is the vector of the nodal displacement, $\boldsymbol{R}$ is the vector of the nodal load. Zero viscosity, $\mathbf{C} = \mathbf{0}$, is considered in the paper. Eq. (1) has the same form for homogeneous materials as well as for inhomogeneous materials including composites with a piecewise constant variation of material properties and functionally graded materials with a continuous variation of material properties.

Due to the space discretization, the exact solution to Eq. (1) contains the numerical dispersion error; e.g., see [15,19,22–27] and many others. Therefore, even the exact time integration of Eq. (1) may lead to inaccurate results due to the space-discretization error. This can be seen for the problems with impact loadings for which large spurious oscillations may appear even for fine meshes in space; e.g., see [14–20]. In our papers for homogeneous materials [14–20] we have suggested several numerical techniques for the reduction of the numerical dispersion error of linear finite elements as well as for the accurate time integration of Eq. (1) without spurious oscillations under high-frequency and impact loadings. In this paper we will extend these techniques for wave propagation in inhomogeneous materials including composites and functionally graded materials.

In the 1-D case, we will use the standard approach with linear 2-node finite elements for which the local consistent mass matrix $\boldsymbol{M}_m^{cons}$ and the stiffness matrix $\boldsymbol{K}_m$ for the mth finite element used in Eq. (1) are given as ($m = 1, 2, 3, \ldots, n$ where n is the total number of finite elements):

$$\begin{aligned}\boldsymbol{M}_m^{cons} &= A\int_0^{dx_m} \rho(s)\boldsymbol{N}^T(s)\boldsymbol{N}(s)ds \\ &= A\int_0^{dx_m} \begin{pmatrix} \left[1 - \frac{s}{dx_m}\right]^2 \rho(s) & \left[\frac{s}{dx_m} - \left(\frac{s}{dx_m}\right)^2\right]\rho(s) \\ \left[\frac{s}{dx_m} - \left(\frac{s}{dx_m}\right)^2\right]\rho(s) & \left(\frac{s}{dx_m}\right)^2 \rho(s)\end{pmatrix} ds,\end{aligned} \tag{2}$$

$$\boldsymbol{K}_m = \int_0^{dx_m} E(s)\boldsymbol{B}^T(s)\boldsymbol{B}(s)ds = \begin{pmatrix} 1 & -1 \\ -1 & 1\end{pmatrix} \frac{1}{dx_m}\int_0^{dx_m} E(s)ds, \tag{3}$$

where $\boldsymbol{N}$ and $\boldsymbol{B}$ are the standard finite element shape and B matrices; A is the cross sectional area in the 1-D case; dx_m is the length of the mth finite element. The local lumped mass matrix $\boldsymbol{D}$ can be obtained from the consistent mass matrix in Eq. (2) by the "row summation" technique. Along with the standard finite element formulations with the consistent and lumped mass matrices, we will use the finite element formulation with reduced dispersion based of a weighted average of the consistent $\boldsymbol{M}^e$ and lumped $\boldsymbol{D}$ mass matrices with the weighting factor γ (similar to that used in [15,19,25,26])

$$\boldsymbol{M}(\gamma) = \boldsymbol{D}\gamma + \boldsymbol{M}^{cons}(1-\gamma). \tag{4}$$

This approach with reduced dispersion will be used below in the 1-D and 2-D cases (see Section 2.2). It is known that for homogeneous materials and uniform meshes, the weighting factor $\gamma = 0.5$ decreases the numerical dispersion error from the second order to the fourth order; see [15,19,25,26]. The value $\gamma = 0.5$ will be also used below in the numerical examples for inhomogeneous materials and non-uniform meshes. The global mass and stiffness matrices in Eq. (1) can be calculated by the standard summation of the corresponding local matrices. In Eqs. (2) and (3) we use the local Cartesian system with the origin at the left node. The difference between the finite element formulations for homogeneous and inhomogeneous materials consists in the fact that density ρ and Young's modulus E in Eqs. (2) and (3) are constant for homogeneous materials and are functions of the space coordinate (the local coordinate s) for inhomogeneous materials. Therefore, in order to increase the accuracy of the results, some approaches were based on a variable density and Young's modulus within a finite element where these quantities in Eqs. (2) and (3) were approximated with the help of the standard finite element shape functions; e.g., see [4,6,10,12]. However, numerical results showed that these modifications do not significantly change the accuracy of numerical results compared with a piecewise constant variation of density and Young's modulus within the domain and the constant values of these parameters (calculated in the center of a finite element) within a finite element; see [4]. In our paper we will use the second possibility with a piecewise constant variation of density and Young's modulus. This will allow us to extend some results for homogeneous materials to inhomogeneous materials (without the modifications of existing computer codes) and to obtain accurate numerical solutions for wave propagation in composite and functionally graded materials; see below.

2.1. Two-stage time-integration technique and its extension to composite and functionally graded materials

The numerical solutions of wave propagation problems under high-frequency and impact loading include spurious high-frequency oscillations due to the large numerical dispersion error for high frequencies. In order to filter the spurious high-frequency oscillations, numerical dissipation (or artificial viscosity) is usually introduced for the time integration of Eq. (1). As we showed in our paper [14], the use of a time-integration method with numerical dissipation (or artificial viscosity) at each time increment leads to inaccurate numerical results for low frequencies as well, especially for a long-term integration. It is also unclear in this case