Composite Structures 117 (2014) 309–315

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Linear static analysis of Functionally Graded Plate using Spline Finite Strip Method

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ARTICLE INFO

Article history:
Available online 10 July 2014

Keywords:
Functionally Graded Plates
Classical Plate Theory
Spline Finite Strip Method
Idealisation techniques

ABSTRACT

Spline Finite Strip Method has been found to be very efficient in the analysis of plates, shells and stiffened plates/shells for their linear static, linear stability, non-linear static, non-linear instability and vibration behaviour. The present work explores the extension of the Spline Finite Strip Method to the analysis of functionally graded material (FGM) plates. Here the modulus of elasticity of FGM plate varies along the thickness direction and the variation is idealised by power, sigmoid and exponential functions. The Poisson's ratio of the FGM plate is assumed to be constant throughout the thickness direction. The analysis is done for moderately thin plates subjected to uniformly distributed, central concentrated and line loads and the deflections and stresses are obtained using the Classical Plate Theory. A comparative study of the three idealisation techniques is also done.

1. Introduction

Functionally graded materials (FGM) are the advanced materials in the family of engineering composites which are developed with a view to tailor the material architecture at microscopic scales. They are made of two or more component characterised by a compositional gradient from one component to the other throughout the thickness. FGM's were initially developed in the late 1980's for use in high temperature applications by a group of Japanese scientists as ultrahigh temperature resistant materials for aircraft, space vehicles and other engineering applications. These advanced materials with engineering gradients of composition, structure and/or specific properties in the preferred direction/orientation are superior to homogeneous material composed of similar constituents [1,7]. The concept of FGM, initially developed for super heat resistant materials to be used in space planes or nuclear fusion reactors, is now of interest to designers of functional materials for energy conversion, dental and orthopedic implants, sensors and thermo-generators and wear resistant coatings. An FGM can be prepared by continuously changing the constituents of multi-phase materials in a pre-determined volume fraction of the constituent material. Due to the continuous change in material properties of an FGM, the interfaces between two materials disappear but the characteristics of two or more materials of the composite are preserved. Subsequently the stress singularity at the interface of a composite can be eliminated and thus the bonding strength is enhanced [2,3]. In view of the wide material variations and applications of FGMs, many research works have already been done for the bending [2,3,5] and buckling analysis [4,6,8–15].

FGMs have heterogeneous microstructure with material properties varying smoothly and continuously in preferred direction. To simplify the complicated heterogeneous microstructure of FGM, different homogenisation schemes are already developed. Power law function (PFGM), exponential function (EFGM) and sigmoid function (SFGM) are widely used to describe the variation of material properties of FGM.

Various well established theories are available for the analysis of isotropic and composite plates which can be extended to FGM plates also. In this work, the Spline Finite Strip Method is used for the bending analysis of FGM plates using Classical Plate Theory. The three idealisation techniques power-law, sigmoid and exponential functions are used to show the variation of Young's modulus along the thickness direction. The Poisson's ratio is assumed to be a constant in all directions. The material properties are varied continuously in the thickness direction according to the volume fraction of the constituent materials. The results are validated using the closed form solution developed by Chi and Chung [2].

2. Spline Finite Strip Method

Although closed form analytical method may be possible in simple cases of idealised structure and loading, various numerical approaches like FEM, CFSM, SFSM, etc. are usually resorted to

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http://dx.doi.org/10.1016/j.compstruct.2014.07.002
0263-8223/

complex systems and loading conditions. The Finite Element Method (FEM) has been extensively used for the analysis of plated structures. The computational requirement of FEM, in terms of storage space and time is very high, especially in linear prismatic members wherein some of the elements have small width. Hence, this method has only limited application in the stability and non-linear analysis of linear prismatic members modelled using plates and shells, especially when iterative non-linear analysis is needed, as in optimum design. The Classical Finite Strip Method (CFSM), on the other hand, allows more efficient modelling of such prismatic members using strips as elements along the length of the member. This method works well for simple boundary conditions (simply supported, clamped, etc.), but fails to effectively deal with complex boundary conditions and partial and concentrated loads, since the trigonometric functions used to model displacements in the longitudinal direction are infinitely continuous. The continuity and discontinuity requirements can be satisfied by replacing the classical trigonometric function by a spline function as is done in Spline Finite Strip Method (SFSM). The Spline Finite Strip Method has been found to be very efficient in the analysis of plates, shells and stiffened plates/shells for their linear static, linear stability, non-linear static, non-linear instability and vibration behaviour [17–19].

The spline function is defined as a piecewise polynomial of nth degree which is smoothly connected to the adjoining spline functions which has n-1 continuous derivatives. There is variety of splines namely natural spline, cardinal spline, basic spline, etc. B_3 spline (cubic basic) is most common and is continuous over only four consecutive sections. The B_3 spline series is a piecewise cubic polynomial, which is an ideal approximation of the bending behaviour. Another property of B_3 spline is its localised behaviour that makes the stiffness matrix highly banded. Owing to this property, incorporating the boundary conditions is easy, and only three splines adjacent to the constraint need to be modified. Equal and unequal spaced spline series have been used by many researchers [16–19] to analyse thin and thick plate structures. The unequal splines are more efficient when the structure is subjected to concentrated loads and reactions, when the support of members are either isolated or at irregular locations and when cut-outs are present. Hence here unequal B_3 splines are used to model the variation of deflection in the longitudinal direction.

3. Problem formulation

The functionally graded material (FGM) can be produced by continuously varying the constituents of multi-phase materials in a predetermined profile. The most distinct features of an FGM are the non-uniform microstructures with continuously graded macro properties. An FGM can be defined by varying the volume fractions using the power-law, exponential or sigmoid function. In the rectangular FGM plate shown in Fig. 1, coordinates x and y define the plane of the plate, and the z-axis originated at the middle surface of the plate is in the thickness direction. The material properties, Young's modulus and the Poisson's ratio, on the upper and lower surfaces are different but are pre-assigned according to the performance demands and the Young's modulus and Poisson's ratio of the plates vary continuously only in the thickness direction (z-axis) i.e., $E = E(z)$, $m = m(z)$. Delale and Erdogan [19] indicated that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus. Hence here Poisson's ratio of the plate is assumed to be a constant. and the variation of Young's moduli is idealised using any one of the three functions i.e. power-law functions (P-FGM), exponential functions (E-FGM), or sigmoid functions (S-FGM).

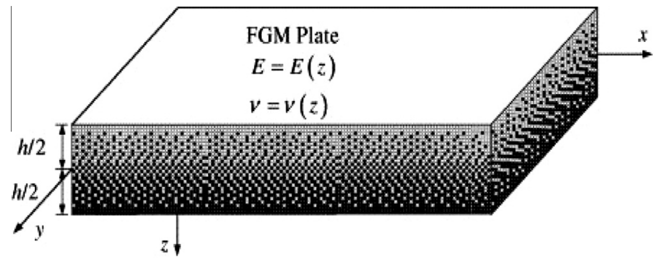

Fig. 1. The geometry of an FGM Plate.

3.1. Power law idealisation

The volume fraction of the P-FGM is assumed to obey a power law function:

$$g(z) = \left(\frac{z + h/2}{h}\right)^p \tag{1}$$

where p is the material parameter and h is the thickness of the plate, the material properties of a P-FGM can be determined by the rule-of-mixture as

$$E(z) = g(z)E_1 + [1 - g(z)]E_2 \tag{2}$$

where E_1 and E_2 are the Young's moduli of the lowest ($z = h/2$) and top surfaces ($z = -h/2$) of the FGM plate, respectively.

3.2. Sigmoid idealisation

The material property variation across the thickness of the FGM plate using the sigmoid function is defined using two power-law functions to ensure smooth distribution of stresses among all the interfaces. The two power-law functions are defined by,

$$g_1(z) = \frac{1}{2}\left(\frac{h/2 - z}{h/2}\right)^p \quad \text{for } 0 \leqslant z \leqslant h/2 \tag{3}$$

$$g_2(z) = 1 - \frac{1}{2}\left(\frac{h/2 + z}{h/2}\right)^p \quad \text{for } -h/2 \leqslant z \leqslant 0 \tag{4}$$

Thus the Young's modulus of the SFGM can be calculated by;

$$E(z) = g_1(z)E_1 + [1 - g_1(z)E_2] \quad \text{for } 0 \leqslant z \leqslant h/2 \tag{5}$$

$$E(z) = g_2(z)E_1 + [1 - g_2(z)E_2] \quad \text{for } -h/2 \leqslant z \leqslant 0 \tag{6}$$

3.3. Exponential idealisation

The exponential function used to describe the material properties of FGMs is

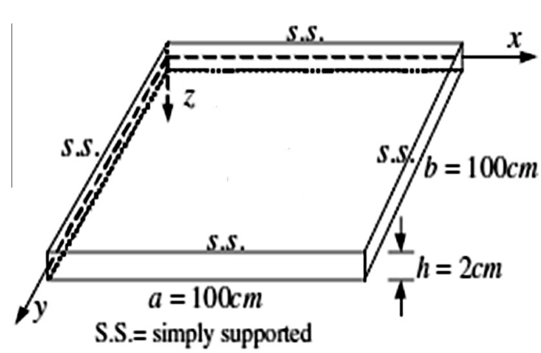

Fig. 2. Configuration of a simply supported square FGM plate.