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Free vibration analysis of sandwich cylindrical panel with functionally graded core using three-dimensional theory of elasticity

A. Alibeigloo [a,*], K.M. Liew [b]

[a] Department of Mechanical Engineering, Faculty of Engineering, Tarbiat Modares University, 14115-143 Tehran, Iran
[b] Department of Civil and Architectural Engineering, City University of Hong Kong, Kowloon, Hong Kong Special Administrative Region

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ABSTRACT

This paper presents an exact three-dimensional free vibration solution for sandwich cylindrical panels with functionally graded core. Material properties of the FGM core are assumed to be graded in the radial direction, according to a simple power-law distribution in terms of volume fractions of the constituents. Poisson's ratio is assumed to be constant. The governing equation of motions is formulated based on the 3D-theory of elasticity and displacement fields are expanded in Fourier series along the in-plane coordinates which satisfy the simply supported edges boundary conditions. The state space technique is used to obtain natural frequencies analytically. Accuracy and convergence of the present approach are examined by comparing the analytical results with the existing values in literature. The parametric study is carried out to discuss the effects of gradient index, geometrical properties such as span angle, facing layers thickness and axial length to mid radius ratio on the frequency behavior of the sandwich panel. The obtained exact solution shows that the FGM core has significant effects on the vibration behavior of sandwich cylindrical panel. This fist known exact solution serves as a benchmark for assessing the validity of numerical methods or two-dimensional theories used to analyses of sandwich cylindrical panels.

1. Introduction

Sandwich panels are important structural elements for many fields of lightweight construction. In conventional sandwich structures, due to the difference between stiffness of the face sheets and the core layer, debonding failure may occur at the interfaces. By using FGM core, resistance of sandwich panels to this type of failure increases.

In recent years, static and free vibration behavior of sandwich panel has been extensively studied by some researchers. Gipson [1] carried out frequency analysis of vibrating sandwich panels with directionally reinforced laminations. Based on the principle of virtual work, Xia and Lukasiewicz [2] analyzed free vibration of viscoelastic, damped sandwich cylindrical panel using the Runge–Kutta method. Free vibration analysis of doubly curved open deep sandwich shells was presented by Singh [3] using the Rayleigh–Ritz method. Nonlinear free vibration of shallow asymmetrical, doubly curved sandwich shell with orthotropic core having different elastic characteristics was presented by Chakrabarti and Bera [4]. Khare et al. [5] used the higher-order shear deformation theory and finite element method to study the free vibration behavior of isotropic, orthotropic and layered anisotropic composite and sandwich laminates. Kashtalyan and Menshykova [6] investigated elastic deformation of sandwich panels with functionally graded core. Based on two dimensional theory, Moreira and Dias Rodrigues [7] carried out static and dynamic analyses of sandwich panel using the finite element method. Rahmani et al. [8] discussed free vibration of composite sandwich cylindrical shell with flexible core using the classical shell theory for the face sheets and elasticity theory for the core layer. Based on the refined three-layered theory, Biglari and Jafari [9] investigated free vibration of doubly-curved sandwich panels with flexible core. Effect of continuously grading fiber orientation face sheets on vibration behavior of sandwich panels with functionally graded core was studied by Sobhani Aragh and Yas [10] using the generalized differential quadrature (GDQ) method. Mohammadi and Sedaghti [11] analyzed free vibration of sandwich cylindrical shell with viscoelastic core using the semi-analytical finite element method. Based on the higher order zigzag theory (HOZT), Kumar et al. [12] analyzed free vibration behavior of laminated composite and sandwich shells using the 2D finite element method. Sobhy [13] investigated vibration and buckling behavior of FGM sandwich plate resting on elastic foundations with various boundary conditions. Yas et al. [14] discussed

* Corresponding author. Tel.: +98 21 82883991; fax: +98 21 82884909.
E-mail address: abeigloo@modares.ac.ir (A. Alibeigloo).

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free vibration behavior of functionally graded nanocomposite cylindrical panels reinforced by single-walled carbon nanotubes using the theory of elasticity and generalized differential quadrature method. Dozio [15] used two-dimensional Ritz method to investigate free vibration behavior of sandwich plate with FGM core layer. By using quasi-3D higher-order shear deformation and a meshless technique, static, free vibration and buckling analysis of isotropic and functionally graded sandwich plates was discussed by Neves et al. [16]. Based on three-dimensional theory of elasticity, author analyzed free vibration behavior of nanoplate, cylindrical shell and cylindrical panel [17–20]. Recently, author [21] used theory of elasticity to carry out free vibration analysis of functionally graded carbon nanotube reinforced composite cylindrical panel embedded in piezoelectric layers.

To our knowledge, three-dimensional free vibration solution of simply supported sandwich FGM cylindrical panel has not yet been investigated. Therefore, this first known solution provides an important benchmark for future assessing the validity of newly developed numerical methods such as meshless methods [22–28] for sandwich cylindrical panels with functionally graded core. In this paper, we will examine the vibration behavior of FGM cylindrical sandwich panel using the Fourier series and state space technique. A few selected example problems of sandwich panels with aluminum/zirconia FGM core layer made of different materials are studied.

2. Theory and formulation

In this study, a simply supported cylindrical sandwich panel composed of metal and ceramic facing sheets and a host FGM core layer is considered. The panel has length L, span angle θ_m, total thickness h, inner and outer radius r_i and r_o, respectively, as depicted in Fig. 1.

The Young's modulus and material density of the FGM core layer are assumed to vary according to the simple power-law along the radial direction

$$E=E_0\left(\frac{r}{r_i+h_m}\right)^{m_1},\quad \rho=\rho_0\left(\frac{r}{r_i+h_m}\right)^{m_2},\quad (r_i+h_m)\leqslant r\leqslant(r_o-h_c) \tag{1}$$

where $m_1=\frac{\ln\frac{E_h}{E_0}}{\ln\frac{r_o-h_c}{r_i-h_m}}$, $m_2=\frac{\ln\frac{\rho_h}{\rho_0}}{\ln\frac{r_o-h_c}{r_i-h_m}}$, E_0, ρ_0, E_h and ρ_h are the Young's modulus and material density at the inner and outer surfaces of the FGM core layer, respectively.

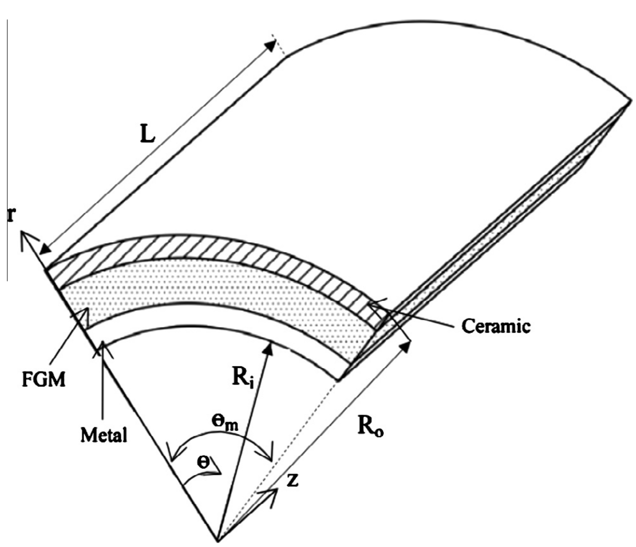

Fig. 1. Geometry of FGM sandwich panel.

The equilibrium equation for the FGM core in cylindrical coordinate can be written in the form:

$$\begin{aligned}
&\sigma_{r,r}+\tau_{zr,z}+\frac{1}{r}\tau_{r\theta,\theta}+\frac{1}{r}(\sigma_r-\sigma_\theta)=\rho\frac{\partial^2 u_r}{\partial t^2}\\
&\tau_{r\theta,r}+\frac{1}{r}\sigma_{\theta,\theta}+\tau_{z\theta,z}+\frac{2\tau_{r\theta}}{r}=\rho\frac{\partial^2 u_\theta}{\partial t^2}\\
&\tau_{rz,r}+\frac{1}{r}\tau_{z\theta,\theta}+\sigma_{z,z}+\frac{\tau_{rz}}{r}=\rho\frac{\partial^2 u_z}{\partial t^2}
\end{aligned} \tag{2}$$

where σ_i, τ_{ij} $(i=r,\theta,z)$ are the normal and shear stress and u_i $(i=r,\theta,z)$ are displacement components along the radial, circumferential and axial directions, respectively.

Stress–displacement relations in linear elasticity are:

$$\begin{aligned}
&\sigma_r=\frac{E(r)}{(1+\nu)(1-2\nu)}\left[(1-\nu)u_{r,r}+\frac{\nu}{r}(u_{\theta,\theta}+u_r)+\nu u_{z,z}\right]\\
&\sigma_\theta=\frac{E(r)}{(1+\nu)(1-2\nu)}\left[\nu u_{r,r}+\frac{1-\nu}{r}(u_{\theta,\theta}+u_r)+\nu u_{z,z}\right]\\
&\sigma_z=\frac{E(r)}{(1+\nu)(1-2\nu)}\left[\nu u_{r,r}+\frac{\nu}{r}(u_{\theta,\theta}+u_r)+(1-\nu)u_{z,z}\right]\\
&\tau_{z\theta}=\frac{E}{2(1+\nu)}\left[u_{\theta,z}+\frac{1}{r}u_{z,\theta}\right]\\
&\tau_{zr}=\frac{E}{2(1+\nu)}[u_{z,r}+u_{r,z}]\\
&\tau_{r\theta}=\frac{E}{2(1+\nu)}\left[\frac{1}{r}u_{r,\theta}+u_{\theta,r}-\frac{u_\theta}{r}\right]
\end{aligned} \tag{3}$$

The simply supported boundary conditions are expressed according to the following relations:

$$u_r=u_\theta=\sigma_z=0 \quad \text{at} \quad z=0,L \tag{4a}$$
$$u_r=u_z=\sigma_\theta=0 \quad \text{at} \quad \theta=0,\theta_m \tag{4b}$$

For free vibration analysis, the conditions on the inner and outer surface boundaries of the sandwich panel are assumed to be traction free

$$\sigma_r=\tau_{zr}=\tau_{r\theta}=0 \quad \text{at} \quad r=r_i,r_o \tag{5}$$

3. Analytical solution

The relations for simply supported boundary conditions, Eqs. (4a) and (4b), are satisfied by the following Fourier series expansion of stress and displacement field:

$$\begin{aligned}
&u_r=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_r\sin(p_m\theta)\sin(p_n z)e^{i\omega t}\\
&u_\theta=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_\theta\cos(p_m\theta)\sin(p_n z)e^{i\omega t}\\
&u_z=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_r\sin(p_m\theta)\cos(p_n z)e^{i\omega t}\\
&\sigma_r=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sigma'_r\sin(p_m\theta)\sin(p_n z)\left(\frac{r}{r_i+h_m}\right)^{m_1}e^{i\omega t}\\
&\sigma_\theta=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sigma'_\theta\sin(p_m\theta)\sin(p_n z)\left(\frac{r}{r_i+h_m}\right)^{m_1}e^{i\omega t}\\
&\sigma_z=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sigma'_z\sin(p_m\theta)\sin(p_n z)\left(\frac{r}{r_i+h_m}\right)^{m_1}e^{i\omega t}\\
&\tau_{rz}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\tau'_{rz}\sigma\sin(p_m\theta)\cos(p_n z)\left(\frac{r}{r_i+h_m}\right)^{m_1}e^{i\omega t}\\
&\tau_{r\theta}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\tau'_{r\theta}\cos(p_m\theta)\sin(p_n z)\left(\frac{r}{r_i+h_m}\right)^{m_1}e^{i\omega t}\\
&\tau_{z\theta}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\tau'_{z\theta}\cos(p_m\theta)\cos(p_n z)\left(\frac{r}{r_i+h_m}\right)^{m_1}e^{i\omega t}
\end{aligned} \tag{6}$$