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Effect of thickness deformation on large-amplitude vibrations of functionally graded rectangular plates

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ABSTRACT

Nonlinear forced vibrations of moderately thick functionally graded (FG) rectangular plates are investigated by considering higher-order shear deformation theories that take into account the thickness deformation effect. The geometrically nonlinear strain–displacement relationships are derived retaining full nonlinear terms in the in-plane and transverse displacements and the three-dimensional constitutive equations are used by removing the assumption of zero transverse normal strain. Both simply supported movable and immovable boundary conditions are considered at the plate edges. The equations of motion are obtained by using multi-modal energy approach. A code based on pseudo arc-length continuation and collocation scheme is utilized for numerical continuation and bifurcation analysis. Results show that higher-order thickness deformation theories yield a significant accuracy improvement for nonlinear vibrations of highly pressurized functionally graded plates.

1. Introduction

Functionally Graded Materials (FGMs) are conceived to be non-homogeneous composites made of a mixture of metal and ceramic with smooth and gradual variation of material properties through the thickness. Of the two constituents that fabricate FGMs, the ductile metal material prevents fracture and the ceramic constituent provides high temperature resistance. Therefore, these materials are capable of withstanding high temperature gradient environments without losing structural integrity, and hence have been proposed as thermal barriers in many engineering applications including space structures and turbine rotors.

Extensive literature surveys on the topic of nonlinear vibrations of plates can be found in the books of Chia [1] and Amabili [2]. A comprehensive review regarding the modelling and analysis of FGM structures have been provided by Birman and Byrd [3]. This review shows that studies dealing with FGM plates are extensive but mostly confined to linear vibration analysis of plates with different boundary conditions and different solution methodologies. To name a few, Reddy and Cheng [4] obtained the frequencies of functionally graded rectangular plates by using a three-dimensional asymptotic approach. Vel and Batra [5] presented a three-dimensional exact solution for the free and forced vibrations of simply supported rectangular plates. Qian et al. [6] used the meshless Petrov–Galerkin method to obtain the natural frequencies of thick FG plates. Batra and Jin [7] used first-order shear deformation theory (FSDT) and finite element method to study the free vibrations of functionally graded anisotropic plates under different boundary conditions.

A complete survey on the topic of nonlinear vibration analyses of FG plates can be found in the book by Shen [8]. The nonlinear vibrations and dynamic response of FG rectangular plates in thermal environments have been investigated by Huang and Shen [9]. Reddy's third-order shear deformation theory was used to obtain the governing equations of motion and a two-step perturbation approach was developed to obtain nonlinear frequencies of plates subjected to temperature variation in thickness direction. Yang et al. [10] and Kitipornchai et al. [11] used a semi-analytical approach that utilized one-dimensional differential quadrature rule and the Galerkin technique to study the effect of piezoelectric actuators and geometric imperfections on nonlinear free vibrations of FG plates, respectively. The effect of piezoelectric actuators on nonlinear frequencies of FG plates have also been studied by Xia and Shen [12] using the same method developed in Ref. [9]. Sundararajan et al. [13] used first-order shear deformation theory and finite-element method to obtain nonlinear fundamental frequencies of FG skewed plates. Woo et al. [14] used mixed Fourier series expansion to obtain the nonlinear frequencies of FG plates with free in-plane boundary conditions. Comparison between Voigt and Mori–Tanaka

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models in predicting the nonlinear vibration response of FG plates has been conducted by Shen and Wang [15]. They found that the difference between the nonlinear frequencies of FG plates obtained via Voigt and Mori–Tanaka models are almost negligible.

Nonlinear vibrations of harmonically excited FG rectangular plates have been studied by Hao et al. [16], Zhang et al. [17], Alijani et al. [18] and Alijani and Amabili [19]. In particular, Hao et al. [16] and Zhang et al. [17] studied the chaotic response of FG plates under simultaneous in-plane and transverse harmonic loads by using models that had simply two [16] and three [17] degrees of freedom. Alijani et al. [18] used the first-order shear deformation theory to study the large-amplitude vibrations and chaotic response of FG rectangular plates with simply supported movable boundary conditions in thermal environments. They found that the effect of volume fraction exponent on the trend of nonlinearity is not significant. Moreover, by plotting the bifurcation diagrams and calculating the Lyapunov exponents, they showed that thermally deformed FG plates are susceptible to quasi-periodic and chaotic vibrations. Alijani and Amabili [19] showed that FG plates under in-plane harmonic excitation lose stability through period-doubling bifurcation and discussed the effect of temperature variations on the on-set of instability.

It can be observed that the nonlinear analyses of FG rectangular plates have been typically based on classical and higher-order two dimensional (2D) theories, which neglect thickness deformation and therefore consider the same transverse displacement for points along the transverse direction (i.e. moving along the plate thickness). In fact, it is believed that due to the strong variation of properties in the thickness direction of FGM structures, the transverse normal strain ε_{zz} could not be discarded. The effect of transverse normal stress σ_{zz} on natural frequencies of multi-layered plates has been shown by Carrera [20]. Carrera and Brischetto [21,22] elaborated the effect of thickness stretching on linear bending and vibrations of isotopic and multi-layered rectangular plates and shells, respectively. In particular, they found that retaining the transverse normal strain ε_{zz} in theories that have linear transverse displacement expansion in the thickness direction could result in a phenomenon known as thickness locking or Poisson locking when three-dimensional (3D) constitutive relations are used. This phenomenon was found to be due to the fact that linear displacement field theories result in a constant transverse normal strain through the thickness that is not consistent with the physical transverse normal strain obtained from plane stress condition. To resolve this issue, they recommended using 2D constitutive relations for classical and shear deformation theories that have constant or linear transverse displacement expansion. Carrera et al. [23] showed for the first time the important role of ε_{zz} in the static response of FG plates and stated that an increase in the order of expansion of in-plane displacements would be meaningless if thickness stretching is discarded in shell and plate theories. Later, the work of Ref. [23] was extended by Cinefra and Soave [24] to study the free vibrations of FG plates by including the effect of thickness stretching. Neves et al. [25,26] also considered the thickness stretching effect and proposed three-dimensional sinusoidal and hyperbolic shear deformation theories to study the static and free vibration problems of FG plates, respectively. Recently, Kim and Reddy [27] presented analytical solutions for bending, buckling and vibrations of FG plates by using a couple-stress third order theory that takes into account the thickness stretching effect. Moreover, Reddy [28] and Reddy and Kim [29] proposed general nonlinear shear deformation theories that include von Kármán type nonlinearities and thickness stretching effect. In particular, by considering a third-order shear deformation theory in which the transverse displacement was expanded up to the 2nd order in terms of thickness coordinate, Reddy [28] used Hamilton's principle to obtain the equations of motion and boundary conditions of FG rectangular plates. Reddy and Kim [29] extended the theory of Ref. [28] by using the modified couple stress theory. New nonlinear higher-order thickness stretching and shear deformation theories for shells of generic shape have been recently proposed by Amabili by using 6 (linear transverse displacement in the thickness coordinate) [30] and 8 parameters (cubic transverse displacement in the thickness coordinate) [31].

As it can be perceived, the role of transverse normal strain (ε_{zz}) and stress (σ_{zz}) on geometrically nonlinear analyses of FG rectangular plates is not evident in the literature. Therefore, in contrast with previous studies, the effect of retaining thickness deformation on nonlinear behaviour of FG plates is elaborated in this paper. In particular, higher-order shear deformation theories with fourth order expansion of in-plane displacements and first, second and third-order expansions of transverse displacement in terms of the thickness coordinate are considered. The geometrically nonlinear strain–displacement relationships are derived retaining full nonlinear terms in the in-plane and transverse displacements and the three-dimensional constitutive equations are used, removing the assumption of zero transverse normal strain. Both simply supported movable and immovable boundary conditions are considered at the plate edges and the equations of motion are obtained via Lagrangian approach. A numerical bifurcation analysis is carried out by using pseudo arc-length continuation and collocation scheme. It is found that the results obtained by keeping only the nonlinear terms of the von Kármán type yield weaker hardening behaviour with respect to results obtained by retaining full nonlinearities in both in-plane and transverse displacements. Moreover, results indicate that the first-order thickness deformation theory gives erroneous linear and nonlinear results if 3D constitutive relations are used. This inaccuracy was found to be due to thickness locking phenomenon which has been resolved by utilizing 2D constitutive relations. Furthermore, it is revealed that the effect of thickness deformation is trivial in predicating the trend of nonlinearity (hardening and/or softening response) of FG rectangular plates unless the plate is highly bent in the pre-vibration state and is subjected to significant thickness deformation. In particular, it is found that the 3rd order thickness deformation theory yields an important accuracy improvement for pressurized functionally graded plates.

2. Nonlinear theory with higher-order thickness stretching and shear deformation

Fig. 1 shows a rectangular plate with in-plane dimensions a and b and thickness h considered in an orthogonal coordinate system (O; x, y, z). The plate is made of functionally graded materials with continuous variation of constituents from metal rich surface at the bottom ($z = -h/2$) to ceramic rich surface at the top ($z = h/2$). The displacements of an arbitrary point of coordinates (x,y) on the

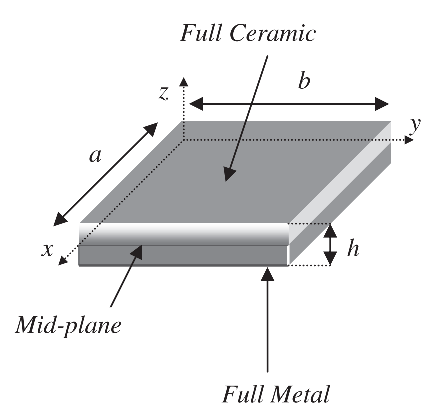

Fig. 1. Geometry of the FG rectangular plate.