Composite Structures 113 (2014) 134–144

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A reduced formulation for the free-wave propagation analysis in composite structures

C. Droz [a,b,*], J.-P. Lainé [a], M.N. Ichchou [a], G. Inquiété [b]

[a] Ecole Centrale de Lyon, 36 Avenue Guy de Collongue, 69134 Ecully Cedex, France
[b] Airbus Helicopters, Aéroport Inter. Marseille-Provence, 13725 Marignane Cedex, France

ARTICLE INFO

Article history:
Available online 19 March 2014

Keywords:
Wave propagation
Composite waveguides
Dispersion curves
Reduction
Wave finite elements

ABSTRACT

This paper presents an improved numerical strategy for the broadband analysis of wave propagation in composite or complex cross-sectional waveguides using the wave finite element method (WFE). Numerical analysis of such structures require highly discretized finite element models and leads to extensive computations. The proposed formulation relies on a projection of the cross-sectional transfer matrices on a reduced set of shape functions associated to propagating waves. Dispersion curves are then predicted only using a reduced number of eigenvectors. The performances and stability of this method are evaluated using the wavenumbers and wave shapes. Validations are provided for a sandwich composite beam and a cylindrical elasto-acoustic waveguide.

1. Introduction

Wave propagation in composite waveguides in a broadband frequency range is widely investigated in automotive and aerospace industry. A waveguide is a structure whose main dimensions exhibit a periodicity or homogeneity in such a way that the propagation of mechanical energy in the main direction (axis of a beam or plane of a plate) is privileged. Waveguide hypothesis can significantly reduce the size of the problem, since the behavior of a single sub-structure of the guide yields the response of the entire structure. Dynamical behavior of such structures is determined by evaluating the set of structural waves propagating through a cross-section [1,2]. One of the major interests of guided waves is their potential for travelling long distances at velocities governed by the dispersion phenomena. The knowledge of these dispersion properties for propagating waves is fundamental for an effective use in engineering, for example in the field of structural health monitoring (SHM).

Numerical prediction of these different waves and their dispersion curves has been extensively studied in last decades. The semi-analytical finite element (SAFE) and wave and finite element (WFE) methods are, among others, very efficient tools for this purpose. In the SAFE method, sinusoidal functions are employed to formulate the displacement field in the direction of propagation. Nevertheless, it is necessary to develop specific semi-analytical elements for each application, which can severely limit its interest for industrial purposes. In order to overcome these limitations, the WFE method combines periodic structure theory (PST) introduced in Mead [3] with a finite element method (FEM).

Therefore conventional finite element software packages can be easily used to compute mass and stiffness matrices of the whole structure. The one-dimensional WFE method was successfully applied to a wide range of waveguides as beams-like structures [4–6], plates [7] and more complex geometries as thin-walled structures [8], tyres [9], pipes [10] and curved layered shells [11].

As the application field of WFE method reaches structurally advanced composite structures, various numerical difficulties can appear, especially for one-dimensional formulation which involves larger cross-sections. Poor-conditioning of the transfer matrix can lead to numerical errors (see Zhong and Williams [12] for alternative formulations), aliasing effects and round-off errors can also appear if cross-section length is not sized carefully. However, for the determination of propagating waves in industrial waveguides involving a large number of degrees of freedom, major obstacle remains the large CPU time needed to solve the eigenproblem. Some numerical issues were investigated for example by Waki et al. [13] and a reduction strategy based on a contributing waves selection was proposed by Mencik [14] to compute forced response of elastic waveguides [15]. Mencik et al. proposed a substructuring technique to compute the appropriate wave motion in multi-layered waveguides. Homogenisation techniques were also investigated [16] to apply the WFE to laminated composites.

* Corresponding author at: Ecole Centrale de Lyon, 36 Avenue Guy de Collongue, 69134 Ecully Cedex, France. Tel.: +33 666233016.

E-mail address: christophe.droz@gmail.com (C. Droz).

http://dx.doi.org/10.1016/j.compstruct.2014.03.017
0263-8223/

Although the aforementioned reduction techniques are interesting to compute forced responses using a reduced number of wave modes, they do not reduce the numerical costs associated with the computation of the wave basis. Therefore, these techniques do not provide a general reduction strategy to compute the propagating waves in elaborated waveguides. This paper presents a method, based on classical formulation of the periodic structure theory, to calculate the dispersion curves of propagating waves for complex cross-section or composite waveguides involving an important number of degrees of freedom. This alternative formulation of WFE method relies on a frequency interpolation of the transfer matrix eigenvectors through a subset of eigensolutions. Thus, the propagating waves can be determined accurately solving a smaller eigenproblem, enabling the application of the WFE method to a wide range of sophisticated cross-sectional waveguide configurations.

The paper is organized as follows. In Section 2, a brief overview of the classical WFE formulation is shown. Section 3 describes the proposed reduction strategy. The model order reduction is formulated for the spectral problem and the strategy of wave interpolation is described; the wave basis is defined next, using a reduced set of propagating waves computed at the cut-on frequencies, associated to the appearance of new propagating waves over the frequency band; a method is then proposed to improve the basis orthogonality and approximate eigenvectors between the cut-on frequencies. Numerical examples are brought in Section 4. The first application concerns a three-layered sandwich beam; both the classical WFE formulation and an analytical low frequency solution described in [17,18] are discussed, and the requirement for a refined FEM of cross-section is highlighted; the reduction strategy is then applied to a detailed FEM model. In the second example, the reduced WFE formulation is extended to an elasto-acoustic problem; dispersion curves are computed for a cavity filled with fluid and compared to the analytical solution.

2. Overview of the WFE

2.1. Free wave propagation in 1D-waveguides

In this section, a formulation of the WFE method is given for free wave propagation in a one-dimensional straight elastic and dissipative waveguide. The structure can be assimilated to N identical subsystems of length d connected along the main direction x. A unit cell of the waveguide is illustrated Fig. 1.

Displacements and forces are written as $\mathbf{q}$ and $\mathbf{f}$, and subscripts 'L' and 'R' denote the left and right edges of a cell. Both edges have the same number n of degrees of freedom. Mesh compatibility is assumed between the N subsystems. The discrete dynamic equation of a cell at frequency ω is given by:

$$(-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K})\mathbf{q} = \mathbf{f} \tag{1}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are the mass, damping and stiffness matrices, respectively. For periodic structures, condensation on the left and right cross-sections of the inner DOF's is required. Introducing the condensed dynamic stiffness operator $\mathbf{D} = -\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}$ and reordering degrees of freedom, equation can be stated as follows:

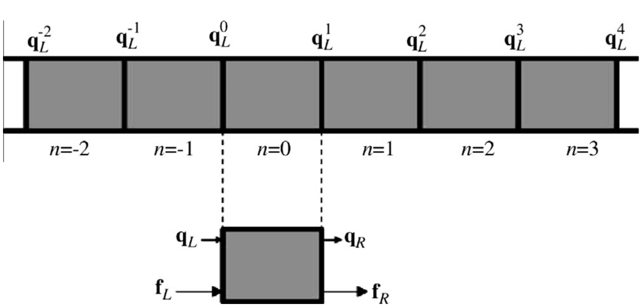

Fig. 1. Illustration of a waveguide and the state vector of a unit cell [1].

$$\begin{bmatrix} \mathbf{D}_{LL} & \mathbf{D}_{LR} \\ \mathbf{D}_{RL} & \mathbf{D}_{RR} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \mathbf{q}_R \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_L \\ \mathbf{f}_R \end{Bmatrix} \tag{2}$$

where $\mathbf{D}_{LL}$ and $\mathbf{D}_{RR}$ are symmetric and $\mathbf{D}_{LR}^t = \mathbf{D}_{RL}$. Denoting $\lambda = e^{-j\kappa d}$ the propagation constant describing wave propagation over the cell length d and κ associated wavenumber, considering force equilibrium $\lambda\mathbf{f}_L + \mathbf{f}_R = 0$ in a cell and invoking Bloch's theorem [19], $\mathbf{q}_R = \lambda\mathbf{q}_L$ into Eq. (2) leads to the following quadratic spectral problem [1]:

$$\left(\lambda_i\mathbf{D}_{LR} + \frac{1}{\lambda_i}\mathbf{D}_{RL} + \mathbf{D}_{LL} + \mathbf{D}_{RR}\right)\mathbf{\Phi}_i^{\mathbf{q}} = 0 \tag{3}$$

where $((\mathbf{\Phi}^q)_i, \lambda_i)_{i=1,\dots,2n}$ stands for the wave modes of the waveguide. The associated eigenvalue problem can be formulated by an appropriate state vector $\mathbf{\Phi}^q = [(\mathbf{\Phi}^q)^t, (\mathbf{\Phi}^f)^t]^t$, leading to a symplectic transfer matrix $\mathbf{T}$.

$$\mathbf{T}\begin{Bmatrix} \mathbf{\Phi}^q \\ \mathbf{\Phi}^f \end{Bmatrix} = \lambda\begin{Bmatrix} \mathbf{\Phi}^q \\ \mathbf{\Phi}^f \end{Bmatrix} \tag{4}$$

with

$$\mathbf{T} = \begin{bmatrix} \mathbf{D}_{LR}^{-1}\mathbf{D}_{LL} & \mathbf{D}_{LR}^{-1} \\ \mathbf{D}_{RL} - \mathbf{D}_{RR}\mathbf{D}_{LR}^{-1}\mathbf{D}_{LL} & -\mathbf{D}_{RR}\mathbf{D}_{LR}^{-1} \end{bmatrix} \tag{5}$$

where eigenvectors represents both nodal displacements and forces associated to a wave mode. The dynamical behavior of the global system can be expressed by expanding amplitudes of incident and reflected waves on a basis of eigenvectors. If the structure is undamped, solutions are divided into propagative waves, whose wavenumbers are real, and evanescent waves for which wavenumbers are imaginary. In dissipative case, complex wavenumbers are associated to decaying waves.

2.2. Computational issues for a complex cross-section

In practice, direct computation of the eigenproblem Eq. (4) can be prone to numerical errors when the meshed cross-section involves a large number of degrees of freedom. Indeed, the transfer matrix T requires to inverse matrix $\mathbf{D}_{LR}^{-1}$ which can be poorly conditioned. To limit this issue, various formulations of the eigenproblem are available, for example:

$$\begin{bmatrix} -\mathbf{D}_{RL} & -(\mathbf{D}_{LL} + \mathbf{D}_{RR}) \\ \mathbf{0} & -\mathbf{D}_{RL} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \lambda\mathbf{q}_L \end{Bmatrix} = \lambda\begin{bmatrix} \mathbf{0} & \mathbf{D}_{LR} \\ -\mathbf{D}_{RL} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \lambda\mathbf{q}_L \end{Bmatrix} \tag{6}$$

However, numerical errors can become serious when eigenvalues are either very large (λ_i) or very small ($1/\lambda_i$), see Zhong and Williams [12] for a detailed discussion. When complex waveguides are considered, an insufficient discretization of the cross-section will produce significant errors, especially for eigensolutions associated to waves whose section shape have a short wavelength, whereas refined meshes exhibit numerous evanescent solutions, thus considerably increases computation time and worsen round-off errors due to the truncation of inertia terms, see Waki [13].

Yet, structures considered in this paper require a high degree of precision due to their geometry, their inner components or for high order wave shapes calculation. In these situations computation time grows exponentially with the number of nodes involved. Classical techniques based on modal basis reduction are not available, since a cross-section boundary conditions are arbitrary for a uniform waveguide or subjected to structural periodicity otherwise. Such an issue is addressed in the next section, introducing projection on a reduced set of shape functions.