Composite Structures 113 (2014) 145–154

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Three-dimensional pyroelectric analysis of a multilayered piezoelectric hollow sphere with imperfect bonding

Chengbin Liu [a], Zuguang Bian [b], Weiqiu Chen [c], Chaofeng Lü [a,*]

[a] Department of Civil Engineering, Zhejiang University, Hangzhou 310058, PR China
[b] Ningbo Institute of Technology, Zhejiang University, Ningbo 315100, PR China
[c] Department of Engineering Mechanics, Zhejiang University, Hangzhou 310027, PR China

ARTICLE INFO

Article history:
Available online 17 March 2014

Keywords:
Piezoelectric hollow sphere
Pyroelectric effect
Imperfect interface
State-space method

ABSTRACT

A three-dimensional pyroelectric analysis is performed for a multilayered piezoelectric hollow sphere with interfacial bonding imperfections using the state-space method. The imperfect interface may not only be mechanically compliant but also be weakly or highly conducting in thermal and electrical fields. A generalized spring-layer model is adopted to describe the imperfect interface for mechanical deformation, while two kinds of models (i.e. lowly conduction and highly conduction) are exploited for the thermal and electric fields. Upon establishing the state-space formulism for each layer, a special but direct treatment of the interfacial conditions is made by introducing the so-called interfacial transfer matrix to facilitate the global analysis. Numerical examples are presented to investigate the effect of bonding conditions on the thermo-electro-mechanical behavior of a multi-layered sphere.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

With the development of smart materials and the associated application technology, spherical shells made of piezoelectric materials have been extensively used. Piezoelectric materials with thermal effect are also known as pyroelectric materials. Pyroelectric materials, usually having temperature-dependent material properties, will produce deformations, stresses, and electric potential when subject to temperature variation. In particular, the effect of thermal field on the behavior of piezoelectric structures has drawn much attention from both engineers and scientists. Mindlin [1] derived a set of two-dimensional theories for thermopiezoelectric crystal plates vibrating at high frequencies. Nowacki [2] and Ieşan [3] presented and discussed some general theorems of thermopiezoelectricity. Chen and Shioya [4] investigated the piezothermoelastic behavior of a pyroelectric spherical shell by employing a displacement method. In the monograph of Carrera et al. [5], a detailed description of the formulation for plates and shells is presented. Due to their great advantages, pyroelectric materials were also widely applied in laminated structures and have been intensively investigated. Shang et al. [6] conducted a thermal buckling analysis of a laminated piezoelectric plate under uniform temperature change. Xu and Noor [7] studied the response of a laminated cylindrical shell to mechanical loading, temperature and electric potential change using the state-space approach. Recently, many researchers have paid their attentions to laminates with interfacial imperfection or damage. The damage may be either caused in the process of fabrication or induced during the service time. Among all weak interface models, the general spring model is the most popular for its convenience. Using the general spring model, Chen et al. [8–14] solved a wide range of static and dynamic problems of piezoelectric beams, plates and cylindrical shells. Brischetto et al. [15] presented a static analysis of multilayered piezoelectric plates by using the principle of virtual displacements. Koutsawa et al. [16] developed a new type of hierarchical FEM to study the static responses of piezoelectric beam structures. But this kind of simplification of the interface is not adequate to reflect all kinds of damages occurring at the interface, especially for heat conduction and dielectricity. Making use of an interface model different from the spring model, Benveniste [17] studied the decay of end effects in heat conduction, considering a sandwich strip with two kinds of imperfect interfaces, i.e. low or high conductivity. Wang and Pan [18] derived exact solutions for simply supported and multilayered piezothermoelastic plates with imperfect interfaces under thermo-electro-mechanical loadings in terms of the pseudo-Stroh formalism, with the imperfect interface described as weakly (highly) thermal conducting, mechanically compliant and weakly (or highly) diectrical conducting. However, little effort has been made to investigate the imperfect laminated sphere, especially for pyroelectric effect.

* Corresponding author. Tel./fax: +86 571 88208473.
E-mail address: lucf@zju.edu.cn (C. Lü).

http://dx.doi.org/10.1016/j.compstruct.2014.03.014
0263-8223/© 2014 Elsevier Ltd. All rights reserved.

In this paper, the state-space method is exploited to investigate the three-dimensional pyroelectric responses of a multilayered piezoelectric spherical shell with imperfect bonding. The imperfect interface here is mechanically compliant and weakly (or highly) thermal (or dielectrical) conducting. For a mechanically compliant interface, we use the general spring model in which the continuous interfacial tractions are linearly proportional to the displacement jump [19]. Two kinds of interface model are adopted to describe thermal and dielectrical conduction. One is a thermally (or dielectrically) weak interface, just like spring model, with the continuous normal heat flux (or the normal electric displacement) proportional to temperature (or electric potential) jump. The other one is a thermally (or dielectrically) strong interface, for which the normal heat flux (or the normal electric displacement) undergoes a discontinuity which is proportional to the surface Laplacian of temperature (or electric potential). For simplicity, the thermally weak interface will be called LC-type (low conductivity), and the thermally strong interface will be called HC-type (high conductivity) [17,18].

First, a second-order homogeneous state equation is established based on the equations governing the heat conduction in the spherical shell. It is solved by a successive use of series expansion technique and matrix theory. A transfer relation is obtained between the thermal state vectors at the inner and outer surfaces of the multilayered shell by using the interfacial transfer matrix at each interface. The thermal field is then exactly determined by incorporating the prescribed surface temperatures into the relation. Second, three displacement functions and two stress functions are introduced to derive a second-order homogeneous and a sixth-order inhomogeneous state equation for the electroelastic field. Exact solutions are obtained via a solution procedure similar to the thermal field. Numerical examples are finally considered and discussed.

2. Basic equations

The basic equations of a spherically isotropic elastic body are well described in the monograph of Ding and Chen [20]. Taking the center of the spherical isotropy as the origin of the spherical coordinates (r, θ, ϕ), we can rewrite the constitutive relations as follows:

$$\begin{aligned}
\Sigma_{\theta\theta} &= c_{11}S_{\theta\theta} + c_{12}S_{\phi\phi} + c_{13}S_{rr} + e_{31}\nabla_2\Phi - \beta_1\Pi,\\
\Sigma_{\phi\phi} &= c_{12}S_{\theta\theta} + c_{11}S_{\phi\phi} + c_{13}S_{rr} + e_{31}\nabla_2\Phi - \beta_1\Pi,\\
\Sigma_{rr} &= c_{13}S_{\theta\theta} + c_{13}S_{\phi\phi} + c_{33}S_{rr} + e_{33}\nabla_2\Phi - \beta_3\Pi,\\
\Sigma_{r\theta} &= 2c_{44}S_{r\theta} + e_{15}\frac{\partial\Phi}{\partial\theta},\\
\Sigma_{r\phi} &= 2c_{44}S_{r\phi} + \frac{e_{15}}{\sin\theta}\frac{\partial\Phi}{\partial\phi},\\
\Sigma_{\theta\phi} &= 2c_{66}S_{\theta\phi},\\
\Delta_\theta &= 2e_{15}S_{r\theta} - \varepsilon_{11}\frac{\partial\Phi}{\partial\theta},\\
\Delta_\phi &= 2e_{15}S_{r\phi} - \frac{\varepsilon_{11}}{\sin\theta}\frac{\partial\Phi}{\partial\phi},\\
\Delta_r &= e_{31}S_{\theta\theta} + e_{31}S_{\phi\phi} + e_{33}S_{rr} - \varepsilon_{33}\nabla_2\Phi + g_3\Pi,
\end{aligned} \tag{1}$$

where $\nabla_2 = r\partial/\partial r$, Σ_{ij} are components of the stress tensor, Φ and Δ_i are the electric potential and electric displacement components, respectively; $\Pi = rT$ with T denoting the temperature change referring to a stress- and electric displacement-free state; c_{ij}, ε_{ij}, e_{ij} and g_3 are the elastic, dielectric, piezoelectric and pyroelectric constants, respectively and

$$\beta_1 = (c_{11} + c_{12})\alpha_{11} + c_{13}\alpha_{33},\ \ \beta_3 = 2c_{13}\alpha_{11} + c_{33}\alpha_{33} \tag{2}$$

where α_{ii} are the thermal expansion coefficients along the radial direction and in the spherical surface. S_{ij} in Eq. (1) is the generalized strain tensor determined by

$$\begin{aligned}
S_{rr} &= rs_{rr} = \nabla_2 u_r,\\
S_{\theta\theta} &= rs_{\theta\theta} = \frac{\partial u_\theta}{\partial\theta} + u_r,\\
S_{\phi\phi} &= rs_{\phi\phi} = \frac{1}{\sin\theta}\frac{\partial u_\phi}{\partial\phi} + u_r + u_\theta\cot\theta,\\
2S_{r\theta} &= 2rs_{r\theta} = \frac{\partial u_r}{\partial\theta} + \nabla_2 u_\theta - u_\theta,\\
2S_{r\phi} &= 2s_{r\phi} = \frac{1}{\sin\theta}\frac{\partial u_r}{\partial\phi} + \nabla_2 u_\phi - u_\phi,\\
2S_{\theta\phi} &= 2s_{\theta\phi} = \frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial\phi} + \frac{\partial u_\phi}{\partial\theta} - u_\phi\cot\theta,
\end{aligned} \tag{3}$$

where s_{ij} are the strain components, u_i $(i = r, \theta, \phi)$ are components of the mechanical displacement. The equations of equilibrium are also rewritten in the following form:

$$\begin{aligned}
&\nabla_2\Sigma_{r\theta} + \csc\theta\frac{\partial\Sigma_{\theta\phi}}{\partial\phi} + \frac{\partial\Sigma_{\theta\theta}}{\partial\theta} + 2\Sigma_{r\theta} + (\Sigma_{\theta\theta} - \Sigma_{\phi\phi})\cot\theta = 0,\\
&\nabla_2\Sigma_{r\phi} + \csc\theta\frac{\partial\Sigma_{\phi\phi}}{\partial\phi} + \frac{\partial\Sigma_{\theta\phi}}{\partial\theta} + 2\Sigma_{r\phi} + 2\Sigma_{\theta\phi}\cot\theta = 0,\\
&\nabla_2\Sigma_{rr} + \csc\theta\frac{\partial\Sigma_{r\phi}}{\partial\phi} + \frac{\partial\Sigma_{r\theta}}{\partial\theta} + \Sigma_{rr} - \Sigma_{\theta\theta} - \Sigma_{\phi\phi} + \Sigma_{r\theta}\cot\theta = 0.
\end{aligned} \tag{4}$$

In the absence of free charge density, the charge equation of electrostatics is

$$\nabla_2\Delta_r + \Delta_r + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\Delta_\theta\sin\theta) + \frac{1}{\sin\theta}\frac{\partial\Delta_\phi}{\partial\phi} = 0 \tag{5}$$

The heat conduction equation is

$$\nabla_2\Theta_r + \Theta_r + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\Theta_\theta\sin\theta) + \frac{1}{\sin\theta}\frac{\partial\Theta_\phi}{\partial\phi} = 0 \tag{6}$$

where $\Theta_i = rq_i$, q_i are the components of heat flux. According to the Fourier's law, we have

$$\Theta_\theta = -\gamma_1\frac{\partial T}{\partial\theta},\ \ \Theta_\phi = -\frac{\gamma_1}{\sin\theta}\frac{\partial T}{\partial\phi},\ \ \Theta_r = -\gamma_3\nabla_2 T \tag{7}$$

where γ_i are the heat conduction coefficients.

3. Determination of thermal field

Consider that a p-ply hollow sphere as shown in Fig. 1 is subjected to given temperature changes at the two spherical surfaces. The temperature field can be solved from Eqs. (6) and (7) and the corresponding boundary/continuity conditions. In this section, we will perform the analysis using the state-space method.

From Eqs. (6) and (7), it is easy to derive the following state equation:

$$\nabla_2\begin{Bmatrix}\Theta_r\\ T\end{Bmatrix} = \begin{bmatrix}-1 & \gamma_1\nabla_1^2\\ -1/\gamma_3 & 0\end{bmatrix}\begin{Bmatrix}\Theta_r\\ T\end{Bmatrix} \tag{8}$$

where Θ_r and T are the state variables for the temperature field, and $\nabla_1^2 = \partial^2/\partial\theta^2 + \cot\theta\partial/\partial\theta + \csc^2\theta\partial^2/\partial\phi^2$ is the two-dimensional

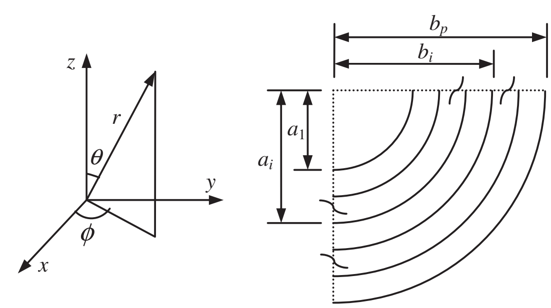

Fig. 1. Spherical coordinates and the geometry of a spherical shell.

Download English Version:

https://daneshyari.com/en/article/251577

Download Persian Version:

https://daneshyari.com/article/251577

Daneshyari.com