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A size-dependent model for bi-layered Kirchhoff micro-plate based on strain gradient elasticity theory

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ABSTRACT

A size-dependent model for bi-layered Kirchhoff micro-plate is developed based on the strain gradient elasticity theory. The governing equations and boundary conditions are derived by using the variational principle. To illustrate the new model, the bending problem of a simply supported bi-layered square micro-plate subjected to constant distributed load is solved. Numerical results reveal that the deflection and axial stress decrease remarkably compared with the classical plate results, and the zero-strain surface deviates significantly from the conventional position, when the thickness of plate is comparable to the material length scale parameters. The size effects, however, are almost diminishing as the thickness of plate is far greater than the material length scale parameters. In addition, the bi-layered plate can be simplified to the monolayer plate as the thickness of one layer is becoming much greater than that of the other layer.

1. Introduction

MEMS (Micro-Electro-Mechanical-System) has been widely used as resonators, biosensors and actuators for its small size, intelligence, conveniently controlling in the field of aerospace, electronics, machinery, medical instruments, civil engineering and so on [1–4]. MEMS devices, according to the geometry and loaded forms, can be simplified to some typical micro-components, such as micro-beam or micro-plate. Since the thicknesses of micro-components are on the order of micron or sub-micro, their mechanical properties are very different from those of macroscopic devices. The mechanical behaviors in micro-structures exhibit obvious size effect, which has been experimentally observed in both metals and polymers [5–7]. The size-dependent behavior cannot be explained by the conventional strain-based theories due to the absence of the internal material length scale parameters. The strain gradient theories have been developed to explain the size dependence of the deformation behavior, in which the material length scale parameters are incorporated into constitutive relations.

According to the deformation metrics used, the strain gradient theories can be classified into couple stress theories and general strain gradient theories. The classical couple stress theory, which uses the higher-order rotation gradients as the deformation metrics, was presented by Mindlin and Tiersten [8] and Toupin [9]. This theory includes two higher-order material constants in addition to the conventional Lame constants. Yang et al. [10], introducing a higher-order equilibrium condition, developed the modified couple stress theory with only one higher-order material parameter. The general strain gradient elasticity theory including five higher-order material constants was firstly proposed by Mindlin [11], in which only the second-order deformation gradients (first-order strain gradients) are included as additional deformation metrics. Also, by using a new set of higher-order metrics and applying the higher-order equilibrium condition, Lam et al. [5] modified the general strain gradient theory and reduced the number of independent higher-order material parameters from five to three. In addition, the simple model with only one additional material constant in the strain gradient elasticity was proposed by Aifantis [12].

In order to explain the size effects in micro-structures, various strain gradient elasticity theories have been used by researchers to develop strain gradient beam and plate theories. For example, the classical couple stress theory has been employed by Anthoine [13] to establish the bending model of a circular cylinder. Park

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and Gao [14] proposed a Bernoulli–Euler beam model based on the modified couple stress theory. The strain gradient elasticity theory has been used by Kong et al. [15] to construct the formulation of a Bernoulli–Euler beam model. By employing the same strain gradient theory, Wang et al. [16] developed a Timoshenko beam model to analyze its static bending and free vibration. For micro-plate, a size-dependent model for the static analysis of Kirchhoff plate with arbitrary shape was presented by Tsiatas [17] based on the modified couple stress theory. Ke et al. [18] and Jomehzadeh et al. [19] employed the same couple stress theory to study the free vibrations of Mindlin micro-plate and Kirchhoff micro-plate, respectively. Based on the strain gradient elasticity theory, a Kirchhoff plate model was developed by Ashoori Movassagh and Mahmoodi [20] and Wang et al. [21].

All above researches are aimed at monolayer micro-components. However, the micro-components are usually bilayered or multilayered structures due to their special micro-machining technology, such as physical and electrochemical depositions [22–24]. Hence, it is essential to develop similar size-dependent models highlighting the laminated micro-components. Zhang et al. [25] studied elastic bending problems of bi-layered micro-cantilever beams subjected to a transverse concentrated load based on the Aifantis' strain gradient elasticity theory. A size-dependent bi-layered microbeam model was developed by Li et al. [26] employing the strain gradient elasticity theory. Researchers further extended the isotropic modified couple stress theory to anisotropic modified couple stress theory and employed this theory to analyze the bending and free vibration of composite laminated beam and plate. Khandan et al. [27] reviewed the development of composite laminated plate theories from very basic classical laminated plate theory to more complicated and higher-order shear deformation theory. The first order shear deformation theory with constant transverse shear stress was proposed by Mindlin [28] and Reissner [29]. Reddy [30] presented a third-order shear deformation theory accounting for parabolic distribution of the transverse shear strains through the thickness of the plate. A model of composite laminated beam based on the global–local theory for new modified couple stress theory was developed by Chen and Si [31]. Roque et al. [32] used the modified couple stress theory to study the bending of simply supported laminated composite Timoshenko beams subjected to transverse loads. The models for composite laminated Reddy beam [33] and plate [34] were developed by Chen et al. employing the modified couple stress theory, respectively. Moreover, for functionally graded beam and plate, Asghari et al. [35], Akgoz and Civalek [36], Reddy and Berry [37], Sahmani and Ansari [38] investigated the static bending and free vibration of FGM micro-beams and micro-plates based on the modified couple stress theory.

In this paper, the bi-layered micro-plate model is developed based on the strain gradient elasticity theory proposed by Lam. The governing equations and boundary conditions are derived by using the variational principle. To illustrate the new model, a boundary value problem of simply supported bi-layered micro-plate is solved. The influences of thicknesses of two layers on the deflection are analyzed. The size effects on deflection, axial stress and location of zero-strain surface are discussed.

2. Size-dependent bi-layered Kirchhoff micro-plate model

2.1. Strain gradient elasticity theory

Lam et al. [5] developed a strain gradient elasticity theory with three independent material length scale parameters. In this theory, the dilatation gradient tensor γ_i, the deviatoric stretch gradient tensor $\eta_{ijk}^{(1)}$ and the symmetric rotation gradient tensor χ_{ij}^{s} are introduced except the classical strain tensor ε_{ij}. These deformation measures are defined as

$$\varepsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i), \tag{1}$$

$$\gamma_i = \partial_i \varepsilon_{mm}, \tag{2}$$

$$\eta_{ijk}^{(1)} = \frac{1}{3}(\partial_i \varepsilon_{jk} + \partial_j \varepsilon_{ki} + \partial_k \varepsilon_{ij}) - \frac{1}{15}[\delta_{ij}(\partial_k \varepsilon_{mm} + 2\partial_m \varepsilon_{mk}) + \delta_{jk}(\partial_i \varepsilon_{mm} + 2\partial_m \varepsilon_{mi}) + \delta_{ki}(\partial_j \varepsilon_{mm} + 2\partial_m \varepsilon_{mj})], \tag{3}$$

$$\chi_{ij}^{s} = \frac{1}{2}(e_{ipq}\partial_p \varepsilon_{qj} + e_{jpq}\partial_p \varepsilon_{qi}), \tag{4}$$

where u_i is the displacement vector, ∂_i is the differential operator, ε_{mm} is the dilatation strain, δ_{ij} is the Kronecker symbol and e_{ijk} is the alternate symbol.

For the isotropic linear elastic material, the strain energy density w_0 is given as

$$w_0 = \frac{1}{2}\lambda\varepsilon_{ii}\varepsilon_{jj} + \mu\varepsilon_{ij}\varepsilon_{ij} + \mu l_0^2 \gamma_i \gamma_i + \mu l_1^2 \eta_{ijk}^{(1)} \eta_{ijk}^{(1)} + \mu l_2^2 \chi_{ij}^{s} \chi_{ij}^{s}, \tag{5}$$

where λ and μ are the Lame constants, l_0, l_1 and l_2 are the independent material length scale parameters associated with the dilation gradients, deviatoric stretch gradients and symmetric rotation gradients, respectively.

2.2. Governing equation and boundary conditions

Consider a bi-layered rectangular elastic micro-plate subjected to a static transverse load $q(x,y)$ distributed in the x–y plane as shown in Fig. 1. The length and width of the plate are a, b, and the thicknesses of the lower and upper layers are h_1 and h_2, respectively. The properties of materials are $E_{(1)}$, $\upsilon_{(1)}$, $l_{0(1)}$, $l_{1(1)}$, $l_{2(1)}$ and $E_{(2)}$, $\upsilon_{(2)}$, $l_{0(2)}$, $l_{1(2)}$, $l_{2(2)}$, where E is the Young's modulus, υ is the poisson's ratio and subscripts 1 and 2 in brackets denote the lower and upper layers, respectively. The position of neutral surface is assumed to be deviated d from the interface between two layers.

For the Kirchhoff plate, where x_0–y_0 plane is coincident with the neutral surface, the displacement components are taking as

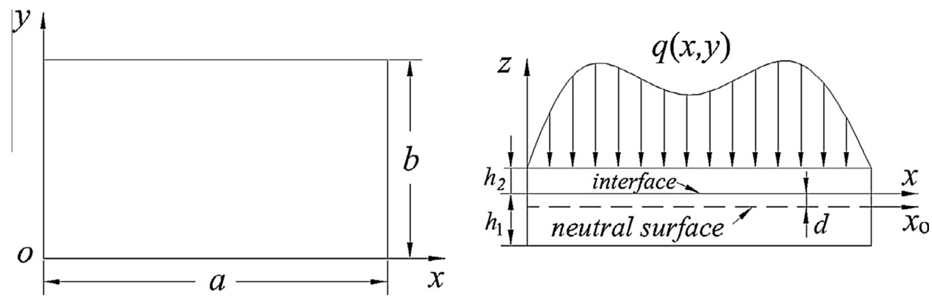

Fig. 1. Schematic of a bilayered micro-plate with distributed load.