Composite Structures 113 (2014) 308–315

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Accurate prediction of free-edge and electromechanical coupling effects in cross-ply piezoelectric laminates

Chao Han [a], Zhangjian Wu [a,*], Zhongrong Niu [b]

[a] School of MACE, The University of Manchester, Manchester M13 9PL, UK
[b] Department of Engineering Mechanics, Hefei University of Technology, Hefei 230009, PR China

ARTICLE INFO

Article history:
Available online 26 March 2014

Keywords:
Piezoelasticity
Laminated plate
Coupling effect
Interlaminar stress
Exact solution
Free-edge

ABSTRACT

3D hybrid analyses on the interlaminar stresses near the free edges of piezoelectric laminated plates are presented on the basis of three-dimensional piezoelasticity. The state space equations for cross-ply piezoelectric laminates subjected to a uniform axial extension are obtained by considering all the independent elastic and piezoelectric constants. With the application of the transfer matrix and recursive solution approach, the equations satisfy the traction-free and open-circuit boundary conditions at free edges and the continuity conditions across the interfaces between material layers. Three-dimensional exact solution is sought and validated by comparing the present analytical results with those from existing approximate and finite element models. The singularities of interlaminar stresses near free edges are observed and the significant electromechanical influence on the free-edge effect is found.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

In recent decades, composite materials have extensive popularity in multifunctional structures. Among various multifunctional structures, the smart materials and structures which can perform sensing, controlling, actuating with distinct direct and converse piezoelectric effects, have been widely used in many applications such as structural vibration control, precision positioning, aerospace and nanotechnology [1]. The piezoelectric structures can be used in the detection and generation of sonar waves and in the ultrasonic transducers for medical imaging. As an emerging application, the development of nanogenerators has demonstrated a possible solution for the design of the self-sufficient power source that directly draws energy from ambient mechanical resources [2] and most recently researchers have engineered the piezoelectric effect into graphene which has the potential to bring the dynamical control to nanoscale electromechanical devices [3].

It is well-established that due to the discontinuity of material properties at the interfaces, a high concentration of a 3D stress field can occur near free edges which will lead to interlaminar failures such as delamination or matrix cracking. Numerous analytical and numerical investigations have been conducted to evaluate this free-edge effect in composite laminates [4–8]. Piezoelectric structures are often made from multi-layered thin films of dissimilar materials in the forms of stacks and a number of interfaces and/or edges will raise concern that they may affect the structural performance of materials and systems [9]. For piezoelectric laminated structures, the intrinsic material properties of piezoelectric laminates can induce the electromechanical coupling effect. The so-called free-edge effect and electromechanical effect may exist simultaneously at the interface of different materials near free edges, which brings more complex phenomena [1].

With the rapid development of piezoelectric laminates, such free-edge and electromechanical coupling effect has drawn the attention of many researchers. By using Fourier transforms to reduce the electro-elastic boundary value problem to the solutions of integral equations, Ye and He [10] solved the problem of electric field concentrations of a pair of parallel electrodes arrayed in one plane. Mannini and Gaudenzi [11] used a multi-layer higher-order finite element approach to investigate a high interlaminar stress concentration between a laminate and distributed piezoelectric actuators near free end. The influence of geometrical and material parameters on the interlaminar stresses in a piezoelectric laminated beam was also assessed by Yang et al. [12] using the finite element model. A notable influence of the electromechanical coupling on the interlaminar stresses and electric fields near the free edges of symmetric laminates under uniaxial extension was detected by Artel and Becker [13] based on the finite element formulation. To delineate the electromechanical behavior of Artel and Becker's model an analytical solution was also developed using a layerwise theory (LWT) by Mirzababaee and Tahani [14]. More-

* Corresponding author. Tel.: +44 1613065792.
E-mail address: Jack.Wu@manchester.ac.uk (Z. Wu).

http://dx.doi.org/10.1016/j.compstruct.2014.03.027
0263-8223/© 2014 Elsevier Ltd. All rights reserved.

over, by employing the improved zigzag LWT Kapuria and Kumari [15] investigated the boundary layer effects in the cross-ply piezoelectric laminates with Levy-type boundary conditions.

In traditional 2D laminate theories, like Reddy's LWT, the thickness-dependent variations for some displacement and electric potential variables are assumed a priori and the boundary conditions at free edges are generally fulfilled only in an integral sense. The finite element method usually provides approximate descriptions for displacements, in-plane stresses and electric variables in the analysis of the piezoelectric laminates with arbitrary geometries and laminations. However, the predicted through-thickness stresses and electric variables cannot satisfy the continuity conditions at interfaces of the laminates sometimes [16] and these results even become unreliable for the free-edge analysis. To the author's knowledge, there is no analytical solution for the cross-ply piezoelectric laminates under a uniform axial extension. In this paper, the state space method [16] is adopted to investigate the free-edge and electromechanical coupling effect in the cross-ply piezoelectric laminates under a uniform axial extension. On the basis of the 3D piezoelasticity, an exact analytical solution that fulfills both mechanical and electric boundary conditions is established by satisfying all the continuous fields, in particular, the interlaminar stress continuity at the interfaces between dissimilar material layers.

2. Formulation of fundamental state space approach

As depicted in Fig. 1 a rectangular piezoelectric laminated plate is subjected to a uniform constant axial strain ε_0 with constant thickness h, width b and infinite length a. The principal elastic directions of the plate coincide with the axes of the chosen rectangular coordinate system and the full three-dimensional piezoelectric–elastic constitutive relations of an orthotropic piezoelectric lamina are given by [17]:

$$\{\sigma\} = [C]\{\varepsilon\} - [e]^T\{E\}, \tag{1}$$

$$\{D\} = [e]\{\varepsilon\} + [\epsilon]\{E\}, \tag{2}$$

where $\{\sigma\}$,$\{\varepsilon\}$, $\{E\}$ and $\{D\}$ are, respectively, stress, strain, electric field, and electric displacement vectors. $[C]$, $[e]$ and $[\epsilon]$ denote elastic, piezoelectric and electric permittivity constants, respectively.

Explicit forms of Eqs. (1) and (2) are given:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}, \tag{3}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}. \tag{4}$$

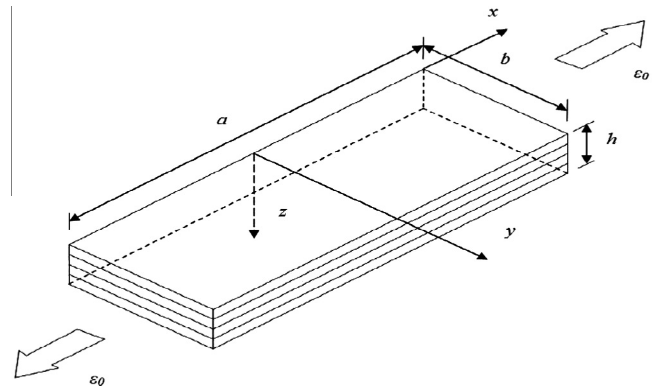

Fig. 1. Geometry and coordinate system of a piezoelectric laminated plate.

Due to the uniform extension ε_0 and infinite length in the x direction, the state variables are independent of the longitudinal coordinate x. As a consequence, the linear strain–displacement relations of elasticity and the electric field-electric potential relations can be written as:

$$\begin{aligned} &\varepsilon_x = \tfrac{\partial u}{\partial x} = \varepsilon_0, \quad \varepsilon_y = \tfrac{\partial v}{\partial y}, \quad \varepsilon_z = \tfrac{\partial w}{\partial z}, \\ &\gamma_{yz} = \tfrac{\partial w}{\partial y} + \tfrac{\partial v}{\partial z}, \quad \gamma_{xz} = \tfrac{\partial u}{\partial z} + \tfrac{\partial w}{\partial x} = 0, \quad \gamma_{xy} = \tfrac{\partial u}{\partial y} + \tfrac{\partial v}{\partial x} = 0, \\ &E_x = -\tfrac{\partial \varnothing}{\partial x} = 0, \quad E_y = -\tfrac{\partial \varnothing}{\partial y}, \quad E_z = -\tfrac{\partial \varnothing}{\partial z}, \end{aligned} \tag{5}$$

where u, v and w represent displacements in the x, y and z directions, respectively. $\varnothing$ is electric potential.

The equilibrium equations of elasticity and the Gaussian law of electrostatics can be expressed as follows:

$$\begin{aligned} &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + f_x = 0, \\ &\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + f_y = 0, \\ &\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0, \\ &\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} - q = 0. \end{aligned} \tag{6}$$

Under the condition of zero electric body charge q and body forces $f_i(i = x, y, z)$ and considering Eqs. (3)–(5), it can be concluded that $\tau_{xz} = 0$, $\tau_{xy} = 0$, $D_x = 0$, and other 9 state variables v, w, σ_x, σ_y, σ_z, τ_{yz}, D_y, D_z, $\varnothing$ are all independent of x. They can be expressed as $v(y,z)$, $w(y,z)$, $\sigma_x(y,z)$, $\sigma_y(y,z)$, $\sigma_z(y,z)$, $\tau_{yz}(y,z)$, $D_y(y,z)$, $D_z(y,z)$ and $\varnothing(y,z)$.

Let $\beta = \frac{\partial}{\partial y}$, from Eq. (5) and the third equations of the matrix in Eqs. (3) and (4), the following relations can be obtained:

$$\frac{\partial}{\partial z}\begin{Bmatrix} w \\ \varnothing \end{Bmatrix} = \begin{bmatrix} k_1\beta & k_2 & k_3 \\ k_5\beta & k_6 & k_7 \end{bmatrix} \begin{Bmatrix} v \\ \sigma_z \\ D_z \end{Bmatrix} + \begin{Bmatrix} k_4 \\ k_8 \end{Bmatrix}\varepsilon_0. \tag{7}$$

By inserting Eq. (7) into Eqs. (3) and (4), the in-plane stresses and electric displacement are presented:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} = \begin{bmatrix} k_9\beta & k_{10} & k_{11} \\ k_{13}\beta & k_{14} & k_{15} \end{bmatrix} \begin{Bmatrix} v \\ \sigma_z \\ D_z \end{Bmatrix} + \begin{Bmatrix} k_{12} \\ k_{16} \end{Bmatrix}\varepsilon_0, \tag{8}$$

$$D_y = k_{17} \cdot \tau_{yz} + k_{18} \cdot \beta\varnothing. \tag{9}$$

The coefficients k_i in matrices and the formulation above are the constants which are only determined by the material properties of the laminate. All the constants are given as follows:

Download English Version:

https://daneshyari.com/en/article/251592

Download Persian Version:

https://daneshyari.com/article/251592

Daneshyari.com