Composite Structures 113 (2014) 382–395

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

# Effects of material uncertainty in the structural response of metal foam core sandwich beams

Carla Beckmann *, Jörg Hohe

*Fraunhofer-Institut für Werkstoffmechanik IWM, Wöhlerstr. 11, 79108 Freiburg, Germany*

ARTICLE INFO

*Article history:*
Available online 27 March 2014

*Keywords:*
Uncertainties
Probabilistic analysis
Homogenization
Random fields
Solid foams
Sandwich structures

ABSTRACT

The present study is concerned with a numerical analysis of the uncertainties in the response of foam core sandwich structures caused by the uncertain microstructure of the core material. In the sense of an integrated computational materials engineering type approach, a three-step procedure is proposed. In the first step, the effective material properties of the core material are computed. For this purpose, a probabilistic homogenization procedure is adopted for the prediction of all elastic constants together with their probability distributions as well as all spatial correlations and interrelations between the individual parameters. In the second step, a random field model for the core material is derived, based on the homogenization results obtained in the first step. The random fields are found to reproduce all stochastic features of the uncertain core material in a proper manner. Using the random field description as an input for a Monte-Carlo-type structural analysis, the uncertainties in the stiffness and strength of sandwich structures are computed in the third step. The capabilities of the proposed procedure are demonstrated using the example of a single edge clamped sandwich beam under transverse loads.


## 1. Introduction

Sandwich structures consisting of two thin, high density face sheets and a thick core consisting of a low density material are important elements in many fields of modern lightweight construction. Whereas the classical field for application of the sandwich principle is the aerospace sector, sandwich structures are found today in many other technologica sectors as well (e.g. Vinson [16], Zenkert [19]). Examples are the wind energy sector, the naval sector or the rail and automotive sectors where an increasing demand for lightweight constructions derives from the limitation of the natural ressources.

Within the principle of sandwich construction, the face sheets carry all in-plane and bending loads whereas the core keeps the face sheets at their desired distance and carries the transverse normal and shear loads. Typical face sheet materials are thin metal sheets or composite laminae. The core usually consists of a weak, low density material. Balsa wood and honeycomb structures are common core materials for high-performance sandwich structures in the aerospace and wind energy sectors. Nevertheless, although their structural performance does not reach the performance of honeycomb structures, solid foams are becoming increasingly popular as core materials for sandwich structures in the transport sectors due to the demand for more complex shapes. Here, the advantage of solid foams is the fact that they can easily be processed to any required shape. Hence, solid foams provide an economically reasonable lightweight solution with a reasonably good strength and stiffness-to-weight ratio. Furthermore, foam core sandwich panels in general have inherently good thermal and acoustic insulation properties and may feature an improved impact and crash resistance.

The main disadvantage of solid foams is their irregular, random microstructure leading to a distinct scatter and uncertainty in the effective properties for this class of materials [13,5]. Especially core structures consisting of metallic foams with large cell sizes suffer from this problem, since their cell size as the relevant microstructural length scale is not sufficiently far below the core thickness as the smallest relevant length scale on the macroscopic level of structural hierarchy [12]. In this case, a probabilistic assessment of the structural response might be advantageous rather than using a classical deterministic approach with large safety factors neglecting the uncertainty.

In this context, the determination of the stochastic material properties for the core material is a crucial point. A proper random field model for the core layer does not only require the mean effective properties together with the variance or standard deviation. Preferably, it should rely directly on the respective probability

* Corresponding author. Tel.: +49 761 5142 458; fax: +49 761 5142 401.
*E-mail address:* carla.beckmann@iwm.fraunhofer.de (C. Beckmann).

http://dx.doi.org/10.1016/j.compstruct.2014.03.030
0263-8223/

distributions, especially in the case of strength analyses where the upper and lower tails of the probability distributions are the relevant ranges of the material properties. The random field should account for all correlations between the different material properties as well as for their spatial correlations and the decay of these correlations with increasing distance. A proper experimental determination of all of the required input parameters is a challenging task, requiring a large amount of experimental efforts. To avoid this effort, a numerical determination of the input parameters for the macroscopic random field analysis on the structural level might be complemented with a probabilistic homogenization analysis of the foam on the microscopic level, providing the respective input properties for the structural analysis.

Regarding the numerical determination of the effective material properties for solid foams, numerous studies are available in literature. The pioneering work has been published by Gent and Thomas [7] in 1963 using an idealized microstructural model based on periodic unit cells. Similar studies based on different idealized microstructural models have been performed, among others, by Gibson and Ashby [8,9] or Christensen [3]. In general, these idealized periodic models as well as further studies based on Kelvin's tetrakaidecahedral foam model [14] provide reasonable approximations of the mean effective stiffness properties for solid foams. Nevertheless, due to their deterministic nature, they are not able to predict the scatter to be expected in the effective material response. For this purpose, a number of stochastic approaches for the numerical prediction of the effective properties have been provided. Among these studies, only the contributions by Kanaun and Tkachenko [10], van der Burg et al. [15] and Zhu et al. [20] based on Voronoï [17] models for the foam microstructure in two and three dimensions should be mentioned. All of these studies are based on the analysis of large scale, statistically representative volume elements. Typically, volume elements consisting of several hundreds of cells are required to obtain stable results.

Since the spatial dimensions of volume elements of this size reach or even exceed the core thickness of typical sandwich structures, the homogenization results based thereon have only limited relevance for the analysis of this type of structure. To avoid this problem, enhanced stochastic homogenization procedures have recently been provided by the present authors [1,2]. In these approaches, the homogenization is performed on a set of small scale testing volume elements. The testing volume elements are not representative for the entire microstructure on their own, however, the entire set of testing volume elements is representative in a stochastic manner. Whereas the former contribution is based on a repeated random generation of small scale volume elements, the latter study [2] employs testing volume elements defined as subsets of a large scale, statistically representative volume element. Hence, this procedure is also able to re-capture all spatial correlations between the effective material properties at neighboring positions.

The present study is concerned with an integrated computational materials engineering type approach for the numerical analysis of sandwich structures with a disordered core material. By this means, the effects of the geometric and topological uncertainty of the microstructure of the core material on the uncertainty in the macroscopic response of the entire sandwich structure can be assessed in a direct manner. The proposed approach consists in a three-step procedure. In the first step, a stochastic homogenization analysis of the foam core material is performed adopting the methods proposed in an earlier study [2]. In a second step, a method for generation of random fields with similar stochastic properties to be used in conjunction with the macroscopic finite element analysis is applied. The third step consists in the stochastic finite element analysis of the considered sandwich structure using a Monte-Carlo type approach in conjunction with a stochastic evaluation of the results. In an example analysis considering a single edge clamped sandwich beam, distinct effects of the microstructural disorder are observed on the strength of the sandwich panel, whereas only minor effects on the (integral) stiffness of the beam are obtained.

## 2. Numerical determination of foam properties

### 2.1. Homogenization process

In the first step of the proposed integrated computational approach for determination of the uncertainties in the structural response of sandwich panels caused by a microstructural disorder of their core material, a stochastic homogenization analysis of the solid foam is performed. For this purpose, the method proposed by the present authors in an earlier study is adopted. Full details on this approach can be found in the original contribution [2]. For sake of completeness, only a brief outline is presented in this section in order to outline the approach and to provide some necessary definition for the further developments.

The stochastic homogenization procedure is based on the concept of the representative volume element. Hence, consider a body $\Omega$ according to Fig. 1 which is bounded by an external boundary $\partial\Omega = \partial\Omega^{u} \cup \partial\Omega^{t}$ with prescribed displacements $u_i = u_i^0$ on $\partial\Omega^{u}$ and prescribed tractions $t_i = \sigma_{ij} n_j = t_i^0$ on $\partial\Omega^{t}$. Furthermore, the body might be loaded by distributed prescribed body forces $f_i = f_i^0$. For its macroscopic analysis, the body $\Omega$ is to be substituted with a similar body $\Omega^*$ with external boundaries $\partial\Omega^{u*}$ and $\partial\Omega^{t*}$ and similar prescribed boundary conditions. The body $\Omega^*$ is assumed to consist of a quasi-homogeneous, "effective" medium with yet unknown properties which has to be equivalent to the cellular microstructure of the original body $\Omega$.

In order to determine the material response of the substitute body $\Omega^*$, representative volume elements $\Omega^{\mathrm{RVE}}$ and $\Omega^{\mathrm{RVE}*}$ of the two opposing bodies are considered. The material response of the volume element $\Omega^{\mathrm{RVE}*}$ consisting of the effective medium have to be chosen such that the response of the two volume elements is equivalent. Within the homogenization procedure proposed by Beckmann and Hohe [2], the mesoscopic equivalence is defined by the usual conditions

$$\varepsilon_{ij} = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE}}} \varepsilon_{ij}^{\mathrm{micro}} \, \mathrm{d}V = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE}*}} \varepsilon_{ij}^{\mathrm{micro}*} \, \mathrm{d}V = \varepsilon_{ij}^* \tag{1}$$

$$\sigma_{ij} = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE}}} \sigma_{ij}^{\mathrm{micro}} \, \mathrm{d}V = \frac{1}{V^{\mathrm{RVE}}} \int_{\Omega^{\mathrm{RVE}*}} \sigma_{ij}^{\mathrm{micro}*} \, \mathrm{d}V = \sigma_{ij}^* \tag{2}$$

that the volume average of the microscopic stress components $\sigma_{ij}^{\mathrm{micro}}$ for the two volume elements has to be equal, provided that both volume elements are subjected to strain states where the local strain components $\varepsilon_{ij}^{\mathrm{micro}}$ are be equal in an volume average sense. In order to determine the effective material constants, the representative volume element $\Omega^{\mathrm{RVE}*}$ is deformed by a number of independent reference deformation states and the corresponding local stresses are computed. Its effective material properties are chosen such that Eqs. (1) and (2) are satisfied contemporarily for all reference strain states considered.

### 2.2. Generation of the computational foam models

The use of the homogenization procedure sketched in Section 2.1 requires the adequate definiton of the microstructure of the volume element $\Omega^{\mathrm{RVE}}$ consisting of the real, microheterogeneous or cellular solid. In the present study, closed cell foams are considered. Its microstructure is generated randomly using a Voronoï tessellation in Laguerre geometry [17,15]in a periodic enhancement. This procedure consists in a random generation of $n$ nuclei with the coordinates $x_i^{nuc(p)}$ within a brick shaped