Composite Structures 113 (2014) 419–428

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Damage onset on tow-placed variable stiffness composite laminates

Hamed Akhavan [a,*], Pedro Ribeiro [a], Marcelo F.S.F. de Moura [b]

[a] DEMec, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
[b] DEMec/INEGI, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal

ARTICLE INFO

Article history:
Available online 28 March 2014

Keywords:
Laminate
Curvilinear fibre
Non-linear vibration
Damage onset

ABSTRACT

Deflection and damage onset of variable stiffness composite laminated (VSCL) rectangular plates with curvilinear fibres, under static and dynamic loads, are investigated. The VSCLs here studied are characterized by the fact that, in each ply, the fibre-orientation angle changes linearly with respect to the horizontal coordinate. A recently developed *p*-version finite element, which follows third-order shear deformation theory (TSDT), is employed. Large deflections are considered, hence, the analysis is in the geometrically non-linear regime. To predict damage onset, Tsai-Wu criterion and an associated damage onset index, described in this paper as a safety factor, are used. It is shown that geometrical non-linearity affects stresses and the location of damage onset. VSCL and CSCL (constant stiffness composite laminated) plates subjected to static, dynamic, and impact loads, are compared.

1. Introduction

For many years, investigations on composite laminated plates essentially focused on composites reinforced with straight fibres, having homogenous orthotropic composite laminas, which one may designate as constant stiffness composite laminated (CSCL) plates. Steering the fibres to make curvilinear paths over the planform of the plate is a tailoring possibility gained by using Automated Tow-Placement Machines [1]. This technology is, maybe for the first time, mentioned in a series of works presented at the International SAMPE Symposiums, including [2–4]. Using curvilinear fibres, the stiffnesses become a function of the position [5]; these non-homogenous, locally orthotropic, layers lead to composites that are termed variable stiffness composite laminated (VSCL) plates, in which modification of load paths offers the possibility to distribute stresses in a more advantageous way [6,7]. Improved structural performances accessible by VSCL plates – like less deflection and smaller likelihood of damage onset under static and dynamic loads – are sought in this paper.

Ref. [8], a review paper on works published before 2010, lists pros and cons of parameterization and optimization algorithms used in the design of VSCLs. Recently, tailoring the fibre orientations, to maximize natural frequency, load carrying capacity or to minimize deflections has attracted a few researchers. Tailoring VSCLs to optimize natural frequency and deflection is essentially related with stiffness (mass also influences the natural frequency, but it is generally assumed that using curvilinear fibres does not affect mass distribution); this is different from maximizing the damage load, which depends on local strength [9]. In [10], the postbuckling first-ply failure response characteristics of VSCLs – modelled in the commercial finite element package Abaqus – are analyzed using a set of physically-based criteria developed in [6]. In [11], the onset of delamination, an important failure mechanism in laminated plates, is evaluated using Abaqus to estimate interlaminar stresses in VSCLs. Delamination initiations are addressed in [12] by studying the response to impacts and the compression after impact, again using Abaqus to model VSCL plates. A multi-objective optimization approach, with a non-dominated sorting genetic algorithm (NSGA-II), is employed in [13] to optimize either the strength around a circular hole with Tsai-Wu failure criterion, or the fundamental frequency, in a VSCL plate. Another design tailoring problem – the pressure pillowing of a fuselage VSCL panel – is addressed in [7], where Abaqus is yet again used with the goal of maximizing the load carrying capacity and the buckling capacity. Refs. [14,15] draw attention to the fact that thick laminates are more likely to experience failure than buckling, because the in-plane failure strains are an order of magnitude smaller than the buckling strains

With knowledge of the published literature on VSCL, the authors believe that a recently developed *p*-version finite element [16,17], with non-linear strain–displacement relations, can be used to properly estimate the damage onset on VSCL plates. The *p*-version finite element method (FEM) has to its advantage a fast convergence rate and the high degree of continuity in the domain [16–19]. The *p*-version finite element with hierarchical basis

* Corresponding author. Tel.: +351 22 508 1723.
E-mail address: hamed@fe.up.pt (H. Akhavan).

http://dx.doi.org/10.1016/j.compstruct.2014.03.038
0263-8223/

Nomenclature

$\mathbf{0}$	a row or column null vector
a, b	length and width of the laminate, i.e., dimensions in x and y directions
c	$4/3h^2$
$\mathbf{f}_i$	vectors of generalized forces ($i = u, v, w, \phi_x, \phi_y$)
f_w	intensity of transverse force components
f^*	interaction term in Tsai-Wu criterion
h	total thickness of the laminate
h_i	thickness of the i^{th} layer
$\mathbf{K}_\mathbf{L}^{ij}, \mathbf{K}_{\mathbf{L}'}^{ij}$	linear stiffness sub-matrices
$\mathbf{K}_{NL}^{ij}$	non-linear stiffness sub-matrices
$\mathbf{M}^{ij}$	mass sub-matrices
$\mathbf{N}^i$	vectors of shape functions ($i = u, v, w, \phi_x, \phi_y$)
n	number of layers
Q_{ij}	elastic properties
$\mathbf{q}_i$	vectors of generalized displacements ($i = u, v, w, \phi_x, \phi_y$)
S_{12}, S_{23}	shear strength of ply ($S_{12} \approx S_{23} = S$)
T_0, T_1	fibre orientations at the centre and edges
t	time
u, v, w	displacements in x, y, z directions
X_t, X_c	longitudinal tensile and compressive strengths of the ply
x, y, z	Cartesian coordinate axes
Y_t, Y_c	transverse tensile and compressive strengths of the ply
α	parameter of proportional damping
$\varepsilon_x, \varepsilon_y$	in-plane strains in Cartesian coordinates x and y
$\varepsilon_1, \varepsilon_2$	in-plane strains in material coordinates
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	shear strains in Cartesian coordinates
$\gamma_{12}, \gamma_{13}, \gamma_{23}$	shear strains in material coordinates
ϕ_x, ϕ_y	rotations around y and x axis
λ	damage onset safety factor
ν_{12}, ν_{21}	Poisson's ratio, $\nu_{21} = \nu_{12}E_2/E_1$
θ	fibre orientation
ρ	volumetric mass density
σ_x, σ_y	in-plane stresses in Cartesian coordinates x and y
$\sigma_1, \sigma_2, \sigma_3$	in-plane stresses in material coordinates
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	shear stresses in Cartesian coordinates x, y and z
$\tau_{12}, \tau_{13}, \tau_{23}$	shear stresses in material coordinates
Ω	surface integration domain

Superscript

0	indicates that a variable is in the mid-plane
T	vector transpose operator

functions, based on a Third-Order Shear Deformation Theory (TSDT) in the linear strain–displacement regime, was already proposed by the authors to analyze the natural modes of vibration of VSCLs [16]. In [17], the authors explored VSCL plates subjected to static loads, to investigate the stress distributions and their dependence on non-uniform stiffness. In this paper, the p-version finite element based on TSDT is applied in the geometrically non-linear regime to VSCL plates, in order to further assess their structural properties. Large deflections and damage onset of VSCL plates, when they are subjected to different static and dynamic loads, as uniform, partial (localized), sinusoidal, and impact loads, are studied.

To predict damage onset in VSCL plates, the well-known and widely used [20–22] stress based Tsai-Wu failure criterion is utilized here, since no stress concentrations are present in the analyzed cases. The aim is to define an in-plane damage onset index and the safety factor for CSCL and VSCL plates. The stress computation with the present model was verified in [17] by comparison with other models and is here further verified by comparison with a 3-D elasticity based analysis.

The paper is organized as follows. In Section 2, the fibre path orientation is given; the section continues with the p-version FEM formulation for a VSCL plate, taking into account geometrical non-linearity, when the plate is under static or dynamic transverse loads; the section ends with Tsai-Wu failure criterion. In Section 3, deflections and indexes of damage onset, in CSCL and VSCL plates subjected to different loadings are presented. Section 4 summarizes the main outcomes of the paper.

2. *p*-Version finite element model for VSCL with curvilinear fibres and damage onset index

Rectangular symmetric laminates will be studied. In Fig. 1, a plate is represented, together with Cartesian coordinates, the origin of which is the plate centroid. The curvilinear fibre paths within the i^{th} lamina are functions of the horizontal coordinate, with the fibre angle in relation to axis x defined as $\theta_i(x) = 2(T_1 - T_0)|x|/a + T_0$. Here, a configuration $[\langle T_0, T_1\rangle^1, \ldots, \langle T_0, T_1\rangle^i, \ldots, \langle T_0, T_1\rangle^k]_{sym}$ represents a $2k$-layered symmetric laminate with fibre angles at the i^{th} layer as $\langle T_0, T_1\rangle^i$.

A p-version finite element, with hierarchical basis functions and following TSDT, is employed to investigate if damage appears on laminates, under the action of various static and dynamic loads. The p-version finite element method has some advantages in comparison with the h-version FEM. In addition to the reasonably fast convergence rate (already mentioned in the introduction), in the present case the laminate is considered as a single element. Hence, not only the implementation of continuity conditions between adjacent elements is omitted, but the degree of continuity in the domain of analysis is increased. The robust convergence of the method is supported by other examples published in linear and non-linear regime [16–18,23–25], as well as, in the linear regime, by theoretical analysis [19].

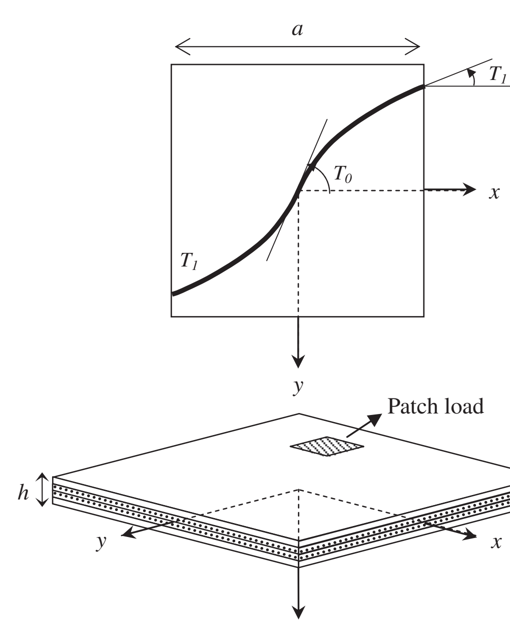

Fig. 1. Configuration of a laminate with curvilinear fibre path in each lamina. A rectangle where a localized distributed load is applied (a "patch load") is also represented. T_0 and T_1 have negative values in this picture.