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Thermal conductivities of iso-volume centre-symmetric honeycombs

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ABSTRACT

The paper describes the equivalent thermal conductivities of iso-volume periodic honeycomb concepts with 3 and 4-connectivities along the in-plane and out-of-plane principal directions. A centre-symmetric 3-connectivity hexagonal layout and a centre-symmetric 4-connectivity cross-chiral configuration are considered. Analytical closed-form solutions of the equivalent thermal conductivities are derived and compared against Finite Element simulations, providing an excellent match. Parametric analyses show the influence of several geometry parameters of the unit cells over the homogenised thermal conductivities along the three principal directions. The analysis shows the capability of tailoring the core geometrical parameters of these iso-volume honeycomb concepts to achieve a desired thermal response in structural sandwich panels or core fillers, for which the coupled thermo-mechanical performance is paramount.

1. Introduction

Structural integrity and heat transfer capabilities are two main design requirements driving the development of multifunctional honeycombs for a variety of aerospace applications. Aeroengine supporting structures, hypersonic flight and re-usable space entry vehicles are examples of operative conditions and applications in which both robust mechanical and thermal performances are required [1]. Among the different thermal protection systems (TPSs) which have been designed in the past decades, sandwich structures employing honeycomb cores have been developed for heat shielding and thermo-mechanical performance [1–4]. The thermal management capabilities of honeycomb cores are primarily driven by the thermal conductivity of the core material, and in second instance by the geometry of the core configuration. The thermal conductivity of periodic and graded cellular configurations have been addressed by different authors in the recent decade, with a focus on the potential benefits that can be achieved in thermo-structural performance by tailoring the geometry of the core configuration [5,6]. General hexagonal honeycomb configurations are characterised by the cell wall aspect ratio α, relative thickness β, and internal cell angle θ [7]. Provided that the honeycomb topology satisfies a minimum internal cell angle requirement for a given aspect ratio [8], it is possible to identify large numbers of honeycomb hexagonal topologies, with positive or negative in-plane Poisson's ratios depending over the value of the internal cell angle, and large variation of their homogenised elastic constants [9]. Between the centresymmetric configurations presented in open literature, the cross-chiral one is worth of notice. Although this tessellation shows an in-plane auxetic behaviour when subjected to uniaxial loading [10,11], Reis and Ganghoffer have recently demonstrated that the structure has a zero in-plane Poisson's ratio and tension-shear coupling resulting in in-plane compliance coefficients $S_{16}, S_{61}, S_{26}, S_{62} \neq 0$ [12].

It is worth of notice that design selection criteria for cores in sandwich structures for multifunctional applications rely on the thermal and mechanical properties of the core, its relative density and gauge thickness of the sandwich panel [13]. Cores are therefore considered as equivalent continuum media, and no explicit relation between the relative scale of unit cells and cellular panels are considered. With the development of truss cores [14,15], the use of Rapid Prototyping to manufacture complex cellular structures [16–18], or even use of custom-based composite manufacturing techniques with relatively large cells [19–21], the issue of the relative scale between cells and panels and the effective available material volume for the load bearing capabilities of the core has to be taken into account. In that sense, an effective way to compare the performance of different honeycomb cores configurations is to use the concept of *iso-volume* tessellation, first introduced by Masters and Evans [9]. The iso-volume honeycomb concept identifies a subset of the potentially infinite honeycomb configurations from hexagonal centresymmetric topologies, allowing to compare auxetic re-entrant and over-expanded configurations with matching

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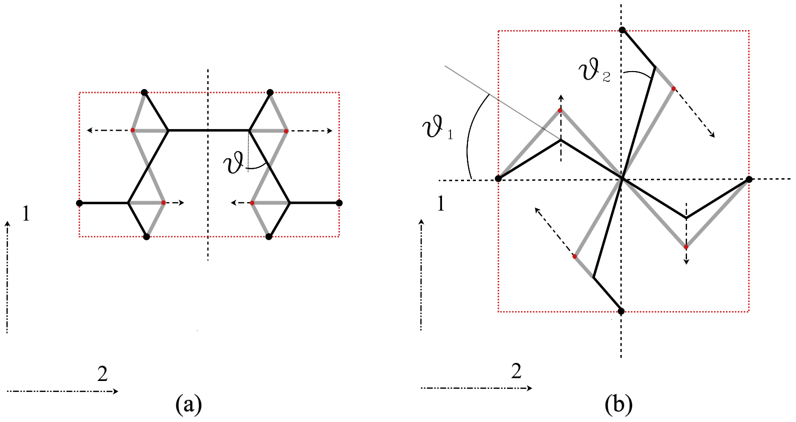
Nomenclature

dx_j	unit-cell projected length along the j-direction (m)
$\zeta = t/dx_2$	ratio between thickness t of the rib and length dx_2 of the iso-volume cell
a, b, c	cross-chiral unit-cell walls (m)
h	centre-symmetric hexagonal horizontal cell-wall (m)
$h_{\mathrm{ref}}, l_{\mathrm{ref}}$	centre-symmetric regular hexagonal horizontal and oblique cell-walls (m)
k_s	thermal conductivity of honeycomb pristine material (W/m K)
k^j_{app}	apparent thermal conductivity along the j-direction (W/m K)
k_{eq}	equivalent thermal conductivity for thermal resistances in parallel or in series (W/m K)
l	centre-symmetric hexagonal oblique cell-wall (m)
l^j_i	i-cell-wall projection length along j-direction (m)
Q	thermal flow (W)
S^j_i	i-cell-wall net section in j-direction for unit width w
t	cell-wall thickness (m)
w	core depth (m)
$\alpha = h/l$	centre-symmetric hexagonal unit-cell aspect ratio
$\beta = t/l$	centre-symmetric hexagonal cell-wall l slenderness ratio
ΔT	temperature change (K)
$\gamma = l/w$	centre-symmetric hexagonal unit-cell gauge ratio
∇T	temperature gradient (K/m)
ρ	honeycomb density (kg/m^3)
ρ_s	density of honeycomb pristine material (kg/m^3)
θ	centre-symmetric hexagonal unit-cell internal angle (°)
θ_1, θ_2	centre-symmetric cross-chiral unit-cell internal angles (°)

unit cell volume to the regular hexagonal shape (internal cell angle of 30° and cell wall aspect ratio of 1 [7]). More recently the use of the iso-volume condition has been applied by [22] in the design of a band-graded auxetic tessellation, where the core geometry describing each band is developed from a baseline regular hexagonal layout. Iso-volume periodic tessellations are sketched in Fig. 1. A baseline geometry contained in a specific volume (black line in Fig. 1) is modified to assume a re-entrant auxetic layout indicated by the red-dot points move along the directions individuated by the dashed arrows (Fig. 1(a)). The new unit cell configuration conserves the original total volume, but provides different engineering elastic constants for the honeycomb. A similar approach can be adopted for the cross-chiral configuration (Fig. 1(b)), in which the baseline layout is provided by the internal angles $\theta_1 = \theta_2 = 45°$, while a demonstration about how different iso-volume cross-chiral configurations do compare is shown in Fig. 2. The two iso-volume configurations are based on the regular centre-symmetric layout, in which the volume of the unit cell is calculated for $\alpha = 1, \theta = 30°$ for the hexagonal tessellation case, and for $\theta_1 = \theta_2 = 45°$ in the cross-chiral case. In particular for the hexagonal centresymmetric configuration, the iso-volume concept allows the structural designer to compare different honeycombs against the regular hexagonal layout, which is the most available core from a commercial point of view.

This work describes the in-plane thermal conductivity and mass properties of the two iso-volume centresymmetric configurations, developing analytical solutions for the in-pane and out-of-plane thermal conductivities of the tessellations. The closed form expressions of the conductivities are formulated versus the core material thermal properties and geometry of the unit cell topologies, and benchmarked against Finite Element models of the honeycombs representing steady-state thermal conduction problems. Although analytical formulations of the thermal conductivities in unusual centresymmetric configurations have been already developed in the past by some of the Authors [23,24], no specific study on iso-volume centresymmetric configuration has been performed. Moreover – to the best of the Authors' knowledge – no analytical and numerical analysis of the thermal conductivities of cross-chiral and iso-volume configurations has been performed to date. As it will be shown in the following paragraphs, hexagonal iso-volume configurations show lower in-plane thermal conductivities in auxetic (negative Poisson's ratio [25,26]) configurations, when compared to classical butterfly auxetic honeycombs. This behaviour is opposite to the one observed previously in open literature [24] and it is a feature worth of, because of the current possibility of building up complex cellular structures composites manufacturing techniques [27].



(a) (b)

Fig. 1. Centre-symmetric iso-volume hexagonal (a) and cross-chiral (b) unit-cells representative of periodic honeycombs. Black lines represent the pristine geometry, gray ones indicate updated geometries following the re-arrangement of the cell-walls.