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Effects of the transfer function evaluation on the impact force reconstruction with application to composite panels

M. Thiene [a,b,*], M. Ghajari [b], U. Galvanetto [a,c], M.H. Aliabadi [b]

[a] CISAS, Università degli Studi di Padova, via Venezia 15, 35131 Padova, Italy
[b] Dep. Aeronautics, Imperial College London, South Kensington Campus, London SW7 2AZ, UK
[c] Dip. Ing. Industriale, Università degli Studi di Padova, via Marzolo 9, 35131 Padova, Italy

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ABSTRACT

The determination of a reliable transfer function for force reconstruction of impacts on composite panels is addressed in the paper. The reconstruction of the impact force history requires the knowledge of the transfer function which relates the response to the contact force. In this paper, a new method to determine the transfer function of a composite plate, instrumented with surface bonded piezoelectric sensors, is proposed. Impact tests are carried out and the data are used to evaluate the transfer function. The force reconstruction results, obtained by using the new transfer function, are compared with the results obtained with the classic approach. Significant improvements are observed in predicting the force history, particularly when large deflections are present; these are quantifiable as an out of plane displacement of the same order of magnitude as the thickness of the plate. The influence of increasing the impact velocity, with the related increase in the contact force, is also studied. The proposed method provides good results over a range of impact velocities. Multiple impacts were also investigated and the method could correctly reconstruct force histories of consecutive impacts.

1. Introduction

The use of composite materials in aeronautical structures has increased significantly in recent years. However, composites have a complex mechanical behaviour, in particular when damage is present. Therefore, a wider application of composite materials in aeronautical structures requires the development of new monitoring techniques. Structural Health Monitoring (SHM) is a multidisciplinary field which includes different topics, from modelling of smart structures to signal processing and developing new sensors [1]. Several different classifications of SHM can be found in literature [1–3]. One classification divides the SHM methods into active and passive [4]. Damage detection techniques, which normally require an active excitation of the system to locate possible defects, are among the active methods. Two examples of the active methods are the vibration-based techniques [5,6] and the lamb wave-based techniques [7–10]. For passive methods, no active excitation is required. The response of the system due to a passive excitation, e.g. impact, is used to characterise the excitation. Impact force reconstruction and determination of impact location lie within these methods.

The problem of impact force reconstruction, commonly referred to as an inverse problem, has been studied by several authors in the last two decades. Inoue et al. [11] investigated the relationship between impact and strain by using the Laplace transform of different sets of training data. In [12,13], the influence of boundary conditions on the transfer function, which represents the mathematical relation between input and output in a mechanical system, as well as the difference between time domain and frequency domain approaches were studied. However, the effects of different impact energies were not considered. Park et al. [14] developed a system identification method to reconstruct and locate impacts on a stiffened panel; the effects of increasing the mass of the impactor were studied. However, the impact velocity, contact force or deflection of the panel was not reported. Hence, the significance of possible geometric nonlinearities was not clear. It was shown in [15,16] that, if the energy (i.e. the mass and/or velocity) of the impact increases, the reconstruction technique based on a single transfer function would fail to reasonably reproduce the time history of the load. Recently, an artificial neural network (ANN) approach has been proposed to both locate and reconstruct impacts, with different energies, on composite plates [15,17–19].

* Corresponding author at: Dep. Aeronautics, Imperial College London, South Kensington Campus, London SW7 2AZ, UK. Tel.: +44 (0)20 7594 5107.

E-mail addresses: m.thiene11@imperial.ac.uk (M. Thiene), m.ghajari@imperial.ac.uk (M. Ghajari), ugo.galvanetto@unipd.it (U. Galvanetto), m.h.aliabadi@imperial.ac.uk (M.H. Aliabadi).

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However a large number of impact data are required in an ANN approach, in contrast to transfer function based methods [11].

The problem of determining a robust transfer function is the most critical issue in the force reconstruction problem. Experimental measurements are not always available and they contain noise, which can adversely affect the quality of transfer functions. Similarly, numerical models always contain simplifications and approximations, particularly in relation to boundary conditions, which can compromise their accuracy and in turn the accuracy of transfer functions. The importance of using an accurate transfer function for structural response reconstruction has been studied in [20,21].

In the present paper, a new method for determining the transfer function of composite structures undergoing large deflections is presented. The proposed method is used to reconstruct the force history of impacts with a relatively large range of velocities. A small number of impact data is used to evaluate the transfer function.

After introducing the mathematical background of the inverse problem and the proposed algorithm, typical examples of limitations of the classic FRF approach are presented. The numerical and experimental models are then introduced, followed by results obtained with the novel approach. A summary of the main contributions and possible future investigation is given in the conclusions.

2. Force reconstruction overview

The dynamics of a linear system excited by a generalised force can be described by a convolution integral as:

$$u(t) = \int_0^t h(t-\tau)p(\tau)d\tau \tag{1}$$

where $u(t)$ represents the response of the system (e.g. displacement, velocity, strains, etc.), $p(t)$ is the force and $h(t)$ denotes the transfer function (i.e. which is commonly defined as the impulse response function). Eq. (1) describes a direct problem, in which the response of the system is calculated from the force and the transfer function. For SHM purposes, a so-called inverse analysis should be performed to determine the force applied to a system, for example an impact on a plate, from the response of the system, such as strain or displacement measured with sensors bonded to the structure. However, to reconstruct the force by using Eq. (1), the transfer function, $h(t)$, should be known. One more problem is the inversion of Eq. (1), which is not straightforward in the time domain. Fourier transform (FT) is utilised to transfer the problem onto the frequency domain. The convolution theorem states that a convolution of two signals in the time domain corresponds to their product in the frequency domain. Hence, Eq. (1) can be rewritten as:

$$U(f) = H(f)P(f) \tag{2}$$

where $H(f)$ is the FT of the impulse response function. For linear systems, $H(f)$ represents the frequency response function (FRF) of the system. Once $U(f)$ and $H(f)$ are known, the FT of the load can be estimated, component by component, with a simple division:

$$P(f_j) = U(f_j)/H(f_j) \tag{3}$$

where subscript j refers to frequency components. The predicted force history can be obtained by applying the inverse FT on $P(f_j)$.

An important element of the solution to the inverse problem, is the transfer function $H(f)$. This function may be estimated for linear systems from a frequency domain analysis on numerical models of the system. However, numerical models always contain simplifications and approximations, particularly in relation to boundary conditions and material properties, which can render the transfer function useless for the inverse problem, especially when several mathematical manipulations of the signals are required [22]. Another important issue is that a classic FRF cannot be applied to nonlinear systems, such as composite panels undergoing large deflections under impact. In this paper, a robust force reconstruction method, suitable for geometrically nonlinear systems, is presented. The mathematical formulation is described in the following section.

3. Force reconstruction algorithm

Consider a system with a discrete impulse response $h(n)$ (where n refers to discrete samples) and a discrete sequence of input values $p(n)$. Eq. (1) can be written as:

$$u(n) = \sum_{k=-\infty}^{\infty} h(n-k)p(k) \tag{4}$$

The autocorrelation functions for the output and input processes are given by:

$$\begin{aligned} s_{uu}[n, n+m] &= E[u[n]u[n+m]] \\ s_{pp}[n, n+m] &= E[p[n]p[n+m]] \end{aligned} \tag{5}$$

where $E[\,]$ denotes the expected value of the variable. Inserting Eq. (4) into Eq. (5) and some manipulations, which can be found in [23], lead to:

$$s_{uu}[m] = \sum_{l=-\infty}^{\infty} c_{hh}[l]s_{pp}[m-l] \tag{6}$$

By using the convolution theorem, Eq. (6) can be moved to the frequency domain:

$$S_{uu}(f) = C_{hh}(f)S_{pp}(f) \tag{7}$$

where S_{uu} and S_{pp} are called the power (or auto) spectra of the output and input respectively. Some studies have used Eq. (7) to determine the transfer function of a system from input/output data [11,24]. However, it is shown below that this equation is not sufficient for determining the transfer function.

Applying the convolution theorem to Eq. (5) leads to:

$$\begin{aligned} S_{uu}(f) &= U(f) \cdot U^*(f) = |U(f)|^2 \\ S_{pp}(f) &= P(f) \cdot P^*(f) = |P(f)|^2 \end{aligned} \tag{8}$$

where * denotes the complex conjugate. By inserting Eq. (2) into Eq. (8), it is found that:

$$\begin{aligned} S_{uu}(f) &= [H(f)P(f)] \cdot [H(f)P(f)]^* = [H(f)P(f)] \cdot [H(f)^*P(f)^*] \\ &= |H(f)|^2 S_{pp} \end{aligned} \tag{9}$$

A comparison between this equation and Eq. (7) indicates that C_{hh} is the module of the transfer function:

$$C_{hh}(f) = H(f)H^*(f) = |H(f)|^2 \tag{10}$$

The module of the transfer function does not contain any information on the phase of the system and therefore, is not sufficient for determining the transfer function. To fully identify the transfer function, the cross correlation function is needed:

$$s_{up}[n, n+m] = E[p[n]u[n+m]] \tag{11}$$

In the frequency domain, the equivalent of the cross correlation is represented by the cross spectrum:

$$S_{up}(f) = U(f) \cdot P^*(f) \tag{12}$$

Inserting Eq. (2) into Eq. (12) and some manipulations lead to:

$$S_{up}(f) = H(f)S_{pp}(f) \tag{13}$$