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Torsional statics and dynamics of nanotubes embedded in an elastic medium

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ABSTRACT

In the present study, torsional statics and dynamics of Carbon Nanotubes (CNTs) embedded in elastic medium are studied using nonlocal elasticity theory. Governing equations of the CNT are obtained using the minimum energy principle. Effects of some parameters like stiffness of elastic medium, geometric properties of CNTs, nonlocal parameter and boundary conditions on the torsional statics and dynamics of CNTs are investigated in detail. Present results can be useful in design of future nano composites, nano electromechanical systems like nano position sensors and linear servomotors.

1. Introduction

Carbon Nanotubes (CNTs) which were discovered by Iijima [1], have extraordinary physical properties like good electrical conductivity, high mechanical strength, etc. With these superior properties, CNTs are used by scientists in several areas like sensor technologies, composites and electromechanical systems.

Two models (The Molecular Dynamic Simulation and the Continuum Model) are used for the modeling of CNTs. The molecular dynamic simulation can be used for small scale structures and short time interval. The continuum model can also be used as an alternative, although it is not size dependent.

Eringen [2,3] proposed the stress gradient type nonlocal continuum theory. He supposed that, stress at a reference point is the functional of the strain field in every point of the continuum. Peddieson [4] has studied static analysis of nano beams by using the nonlocal Euler–Bernoulli model. Sudak [5] used the nonlocal elasticity in column buckling. Wang and Liew [6] analyzed the statics of nano structure with using Timoshenko beam theories. Ghannadpour et al. [7] studied bending, buckling and vibration analysis of the nonlocal Euler beams using Ritz method. Reddy [8] reformulated various beam theories like the Euler–Bernoulli, Timoshenko, Reddy and Levinson, using the nonlocal differential constitutive relations of Eringen. Aydogdu [9] developed a general nonlocal beam theory which contains the former beam theories. Fazelzadeh et al. [10] used a nonlocal anisotropic shell model to study vibration of Single Walled Carbon Nanotubes (SWCNTs) with arbitrary chirality. Zhang et al. [11] analyzed the flexural strength and free vibration of carbon nanotube reinforced composite cylindrical panels. Vibration of CNTs, nanobeams and nanorods are also studied in former studies [12–15].

Recently, wave propagation of SWCNTs and Double Walled Carbon Nanotubes (DWCNTs) are compared with using the nonlocal continuum models and molecular dynamic simulations [16]. Very close results are obtained. Huang et al. [17] studied the behavior of flexural waves traveling in CNTs in a free space and embedded in an elastic matrix.

Torsional vibration of CNTs will be very important for the nano electromechanical devices like linear nano servomotors and bearings in future electrical and mechanical products. Dong et al. [18] investigated nanoscale linear servomotors with integrated position sensing from experimental, theoretical and design perspectives. Hall et al. [19] reviewed major results of advances in fabrication CNTs which allows measuring mechanical and electrical response with applying twisting force. Selim [20] examined the torsional vibration of SWCNTs which subjected to initial compression stress. Gheshlaghi et al. [21] studied torsional vibration of CNTs using a modified couple stress theory. Murmu et al. [22] analyzed the torsional vibration of SWCNT with Buckyball system using the nonlocal elasticity theory. Khademolhosseini [23] investigated the size effects in the dynamic torsional response of SWCNTs by developing a modified nonlocal continuum shell model. Lim et al. [24] developed a new elastic nonlocal stress model and analytical solutions for torsional dynamics of CNTs. Hu et al. [25] made a brief review of vibrations of SWCNTs using nonlocal beam model, nonlocal rod model and MD simulation. Kiani [26] studied the nonlocal Rayleigh beam theory for axially moving SWCNT with simply

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supported ends. Li [27] investigated the torsional static and dynamic behaviors of nanoshafts, nanorods and nanotubes based on a new nonlocal elastic stress field theory. Zhang et al. [11] studied statics and dynamics of carbon nanotube reinforced functionally graded cylindrical panels. Postbuckling of carbon nanotube-reinforced functionally graded cylindrical panels under axial compression using a meshless approach has been investigated by Liew et al. [28]. Large deflection of geometrically nonlinear analysis of carbon nanotube-reinforced functionally graded cylindrical panels has been studied by Zhang et al. [29]. According to author's knowledge torsional statics and dynamics of CNT embedded in an elastic medium has not been considered in the previous studies.

The aim of this study obtaining the governing equations of the CNTs by using the minimum energy principle and the nonlocal stress theory. Then, analyzing the static and dynamic behavior of the CNTs with and without in Elastic Medium. The effects of some parameters like, stiffness of elastic medium, geometric properties of CNTs, nonlocal parameter and boundary conditions of the CNTs on torsional vibrations are studied.

2. Analysis

2.1. Governing equations

A nanotube of length L and diameter d is considered (Fig. 1). The minimum energy principle is used in order to obtain the governing equation of torsional statics and dynamics of nanotubes.

The kinetic energy of CNT can be written as;

$$E_k = \frac{1}{2}\int_0^L \rho I_P \left(\dot{\theta}\right)^2 dx \tag{1}$$

where ρ is the density, I_P is the polar moment of inertia, R_1 and R_2 is the inner and outer radius, θ is the angular displacement of CNT. The Ip is defined as:

$$I_P = \pi \frac{\left(R_2^4 - R_1^4\right)}{2} \tag{2}$$

The Strain Energy due to torsion is defined as:

$$V = \frac{1}{2}\int_0^L GI_P(\theta')^2 dx \tag{3}$$

where G is the shear modulus of CNT. Total virtual work due to torque $T(x,t)$;

$$\delta\overline{W} = \int_0^L \delta\theta\, T\, dx \tag{4}$$

by applying the minimum potential energy principle the following equation can be obtained:

$$\int_0^L \int_{t_1}^{t_2} \left[-\rho I_P \ddot{\theta} + (GI_P\theta')' + T(x,t)\right]\delta\theta\, dx\, dt - \int_{t_1}^{t_2} GI_P\theta'\delta\theta|_0^L dt = 0 \tag{5}$$

From Eq. (5) the governing equations and boundary conditions of the torsional behavior are obtained in the following form:

$$\rho I_P \ddot{\theta} = (GI_P\theta')' + T(x,t) \tag{6}$$

and boundary conditions:

$$GI_P\theta' \quad \text{or} \quad \theta = 0 \tag{7}$$

2.2. Non-local elasticity for CNTs

The nonlocal constitute relation can be given as [9,15];

$$\left(1 - \mu\nabla^2\right)\tau_{kl} = \lambda\varepsilon_{rr}\delta_{kl} + 2G\varepsilon_{kl} \tag{8}$$

where τ_{kl} is the nonlocal stress tensor, ε_{kl} is the strain tensor, λ and G are the Lame constants, $\mu = (e_0a)^2$ is called the nonlocal parameter, a is an internal characteristic length and e_0 is a constant. e_0 value is very important for the validity of nonlocal models. Eringen determined this parameter with matching the dispersion curves based on the atomic models. Wang and Wang made estimation for the SWCNT as $e_0a \leqslant 2$ nm.

For the torsional deformation of uniform CNT, Eq. (8) can be written in one dimensional form:

$$\left(1 - \mu\frac{\partial^2}{\partial x^2}\right)\tau = G\gamma \tag{9}$$

where γ is the shear strain, τ is the shear stress of CNT. The stress resultant due to the shear stress is expressed as:

$$S = \int_A \tau\, dA \tag{10}$$

where A is the cross-section of the CNT, and the torque relation is given as;

$$T = \int_A \tau z\, dA \tag{11}$$

By using the Eqs. (9)–(11), we get the constitute relation as;

$$S - (e_0a)^2\frac{\partial^2 S}{\partial x^2} = GA\gamma \tag{12}$$

$$T - (e_0a)^2\frac{\partial^2 T}{\partial x^2} = GI_P\frac{\partial\theta}{\partial x} \tag{13}$$

If Eq. (13) is inserted into Eq. (6) one obtains

$$\rho I_P\ddot{\theta} = GI_P\theta'' + \mu\left[\rho I_P\left(\ddot{\theta}\right)'' - T''\right] + T \tag{14}$$

Eq. (14) is the governing equation of the CNT for the torsional deformation. If we choose $\mu = 0$ we get classical elasticity equation of torsional deformation.

3. Static case

Now consider a CNT embedded in an elastic medium (Fig. 2). The elastic medium is assumed as torsional springs attached to CNT. The governing equation for the CNT embedded in an infinite elastic medium can be written as;

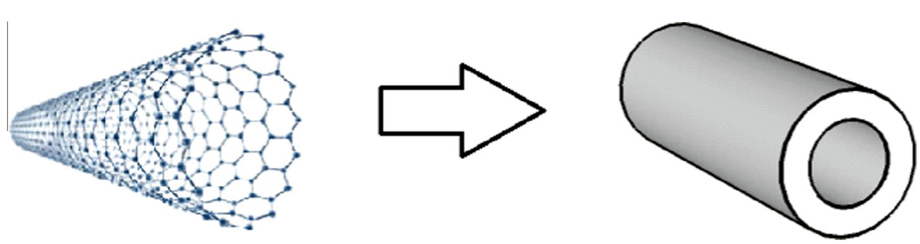

Fig. 1. Carbon nanotube (molecular and continuum models).