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Transverse vibrations of single-walled carbon nanotubes with initial stress under magnetic field

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Dedicated to a distinguished scientist Prof.Dr.Dr.h.c.M.Cengiz Dökmeci (Istanbul Technical University) in honor of him 78th birthday by wishing many more happy years.

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ABSTRACT

This article presents an analytical study, by considering the initial stress presence, for the transverse vibrations of single-walled carbon nanotubes under longitudinal magnetic field. The behavior of single-walled carbon nanotube is modelled as a Timoshenko beam. The explicit solutions are derived for both a combined stress/strain and a combined strain/inertia gradient elasticity theories. A numerical application is carried out for a simply supported single walled carbon nanotube. The numerical results showing the effects of initial stress and longitudinal magnetic field are presented in detail, for the different length scale parameters. This analysis reveals new significant findings for the critical buckling stress and the different capabilities of scale parameters under the magnetic field. This work carried out for the effect of longitudinal magnetic field on the wave propagation in the carbon nanotubes containing the initial stress may be useful particularly in the design and applications of nano electro mechanical systems.

1. Introduction

In recent years, an increasing interest is observed on studying the different magnetic properties of nanotubes, and the vibration and wave propagation of nanotubes under a magnetic field. The studies [1–9] of electronic and transport properties of nano tubes under a magnetic field has attracted considerable interest among the researchers. The effects of longitudinal magnetic field on wave propagation for carbon nanotubes embedded in elastic matrix was addressed in [10] with Eringen's nonlocal elasticity theory. The effects of longitudinal magnetic field on ultrasonic vibration in single-walled carbon nanotubes was investigated in [11] with Eringen's nonlocal elasticity theory. The results of an investigation into the effect of transverse magnetic fields on dynamic characteristics of multi-walled carbon nanotubes were reported in [12]. The effect of longitudinal magnetic field on ultrasonic wave dispersion characteristics of an equivalent continuum structure of single-walled carbon nanotubes embedded in elastic medium was reported in [13] by using nonlocal Euler–Bernoulli beam model. An analytical approach to study the effect of a longitudinal magnetic field on the transverse vibration of magnetically sensitive double-walled carbon nanotube was presented in [14]. The transverse wave propagation within elastically confined single-walled carbon nanotube under a longitudinal magnetic field was investigated in [15] by using nonlocal Rayleigh, Timoshenko and higher -order beam theories. The effects of a longitudinal magnetic field on the vibration of a magnetically sensitive double single walled carbon nanotube system was reported in [16] by using Euler–Bernoulli beam model. The effect of an in-plane magnetic field on the transverse vibration of a magnetically sensitive single-layer sheet was examined in [17] by using equivalent continuum nonlocal elastic plate theory.

As known, temperature change, lattice mismatch or initially external loads often lead to the initial stresses in the carbon nanotubes Although generally the presence of initial stress is notable for the vibration and wave propagation analysis, it is more crucial for the nano scale systems due to their structural properties [18–21], in particularly. To the best knowledge of the author, the investigation of the effect of magnetic field on the transverse wave propagation in the single walled carbon nanotubes having the initial stress is the first time in the open literature. The present article aims to study on the transverse vibrations of the magnetically sensitive single-walled carbon nanotubes with initial stress under the longitudinal magnetic field. In the analysis, the single-walled carbon nanotube is defined as Timoshenko beam. The present explicit solutions are derived by using the combined stress/strain and the combined strain/inertia gradient elasticity theories. In the framework of present analysis, the lower and upper bounds of frequencies are predicted according to two different gradient elasticity models, for the single walled carbon nanotubes which contain the initial stress under the magnetic field effect. As indicated in [22], the electronic and transport properties of the carbon

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nanotubes are extremely sensitive to even if very small distortions. Therefore, new findings and some significant numerical results obtained for the effect of longitudinal magnetic field on the wave propagation in the carbon nanotubes having the initial stress may be helpful in the optimal and rational design of nano-oscillators, nano-sensors and actuators.

2. Theoretical formulation

The basic governing equations of the planar motion under the initial stress and the longitudinal magnetic field can be expressed as follows [23–25]:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} - \sigma_o \frac{\partial^2 u}{\partial x^2} + f_x = \rho \frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial \tau_{xz}}{\partial x} - \sigma_o \frac{\partial^2 w}{\partial x^2} + f_z = \rho \frac{\partial^2 w}{\partial t^2} \tag{2}$$

where σ_{xx} is the axial stress, τ_{xz} is the shear stress, σ_o is the axial initial stress (the sign of σ_o is positive for compressive loading and is negative for tensile loading) z is the normal to the x-axis, ρ is the mass density, u and w are the axial and transverse displacements, respectively, f_x and f_z denote the body forces due to the longitudinal magnetic field. The present analysis is restricted by the plane bending in the xoz plane.

According to Timoshenko beam theory, the axial displacement u, and the transverse displacement w are given by

$$u = z\psi(x,t), \text{ and } w = w(x,t) \tag{3}$$

where ψ is the rotation due to bending alone. The displacement v in the direction y is taken as zero due to planar deformation assumption.

Considering the displacement field (3), the axial strain ε_{xx} and the shear strain γ_{xz} are obtained as follows

$$\varepsilon_{xx} = z\frac{\partial \psi}{\partial x} \tag{4}$$

$$\gamma_{xz} = \psi + \frac{\partial w}{\partial x} \tag{5}$$

On the other hand, the bending moment M and the shear force Q can be defined by the following integrals

$$M = \iint \sigma_{xx} z dA, \text{ and } Q = \iint \tau_{xz} dA \tag{6}$$

where A is the cross-sectional area.

The Lorentz forces due to the longitudinal magnetic field along x, y and z directions are [11]:

$$f_x = 0 \tag{7}$$

$$f_y = \eta H_x^2 \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} \right) \tag{8}$$

$$f_z = \eta H_x^2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z} \right) \tag{9}$$

Considering the displacement field (3) and the planar deformation assumption, Lorentz force in the only z direction reduces to the following form:

$$f_z = \eta H_x^2 \frac{\partial^2 w}{\partial x^2} \tag{10}$$

where η is the magnetic permeability, and H_x is the component in x direction of the longitudinal magnetic field vector exerted on the beam.

For establishing the planar motion equations in terms of M, Q, and Lorentz force are used Eqs. (1), (2), (6), and (10). Hence, multiplying Eq. (1) by zdA and performing integration over the area A, the result is obtained in the following form:

$$\frac{\partial M}{\partial x} - Q - \sigma_o I \frac{\partial^2 \psi}{\partial x^2} = \rho I \frac{\partial^2 \psi}{\partial t^2} \tag{11}$$

where I is the second moment of the area. On the other hand, the integration of Eq. (2) over the area yields

$$\frac{\partial Q}{\partial x} - \sigma_o A \frac{\partial^2 w}{\partial x^2} + \eta A H_x^2 \frac{\partial^2 w}{\partial x^2} = \rho A \frac{\partial^2 w}{\partial t^2} \tag{12}$$

3. The unified stress/strain gradient elasticity model

A unified stress/strain gradient elasticity model used here is a combination of the nonlocal integral model of Eringen [26] and the gradient elasticity model [27]. Then, this unified model [28–30] has been proposed as

$$(1 - l_m^2 \nabla^2)\sigma_{ij} = (1 - l_s^2 \nabla^2)(\lambda \delta_{ij} \varepsilon_{kk} + 2G\varepsilon_{ij}) \tag{13}$$

where l_m and l_s are the material constants in the nonlocal integral and gradient elasticity models, respectively. λ, and G Lame constants, $\nabla^2 \left(= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$ is Laplacian operator and δ_{ij} denotes Kronecker delta.

Considering the exist stress and strain components, according to this unified model (13), the constitutive relations can be expressed as the following partial differential forms:

$$\sigma_{xx} - l_m^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\left(\varepsilon_{xx} - l_s^2 \frac{\partial^2 \varepsilon_{xx}}{\partial x^2} \right) \tag{14}$$

$$\tau_{xz} - l_m^2 \frac{\partial^2 \tau_{xz}}{\partial x^2} = G\left(\gamma_{xz} - l_s^2 \frac{\partial^2 \gamma_{xz}}{\partial x^2} \right) \tag{15}$$

where E is the Young's modulus.

Using Eqs. (4)–(6), the constitutive relations (14) and (15) in terms of the bending moment M and the shear force Q are re-expressed in the following forms:

$$M - l_m^2 \frac{\partial^2 M}{\partial x^2} = EI\left(\frac{\partial \psi}{\partial x} - l_s^2 \frac{\partial^3 \psi}{\partial x^3} \right) \tag{16}$$

$$Q - l_m^2 \frac{\partial^2 Q}{\partial x^2} = KGA\left[\psi + \frac{\partial w}{\partial x} - l_s^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3} \right) \right] \tag{17}$$

where K is the shear correction factor.

The unified nonlocal Timoshenko beam model defined by Eqs. (16) and (17) has been already considered by Elishakoff et al. [31], Challamel [32] and Wu et al. [29]. This unified nonlocal Timoshenko beam model is the generalization of the unified Euler–Bernoulli beam model introduced by Challamel and Wang [33] and Zhang et al. [34].

The second derivative of Q from (12) is written as

$$\frac{\partial^2 Q}{\partial x^2} = \rho A \frac{\partial^3 w}{\partial x \partial t^2} + \left(\sigma_o - \eta A H_x^2 \right) \frac{\partial^3 w}{\partial x^3} \tag{18}$$

Substituting Eq. (18) into Eq. (17), for this unified model the shear force Q is expressed as

$$Q = l_m^2 \left[(N_o - \eta A H_x^2) \frac{\partial^3 w}{\partial x^3} + \rho A \frac{\partial^3 w}{\partial x \partial t^2} \right] + KGA\left[\psi + \frac{\partial w}{\partial x} - l_s^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3} \right) \right] \tag{19}$$