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Nonlinear vibrations of functionally graded Mindlin microplates based on the modified couple stress theory

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ABSTRACT

This paper investigates the size-dependent vibrational behavior of functionally graded (FG) rectangular Mindlin microplates including geometrical nonlinearity. The FG Mindlin microplate is considered to be made of a mixture of metal and ceramic according to a power law distribution. To this end, based on the modified couple stress theory (MCST) and Hamilton's principle, the governing equations of motion and associated boundary conditions are derived. In the solution procedure, the set of nonlinear partial differential equations is discretized through the generalized differential quadrature (GDQ) method. Afterwards, the numerical Galerkin scheme is employed to convert the discretized partial differential equations of motion to the Duffing-type ordinary differential equations. The periodic time differential operators introduced based on the derivatives of a periodic base function are used to discretize differential equations on the time domain. Finally, the pseudo arc-length continuation method is utilized to numerically solve the set of nonlinear algebraic parameterized equations. The effects of the important parameters including material gradient index, length-to-thickness ratio, length scale parameter, and boundary conditions on the vibrational characteristics of rectangular Mindlin microplate are thoroughly discussed.

1. Introduction

Introduction of functionally graded (FG) materials to micro- and nano-structures has profoundly facilitated achieving the most desired micro- and nano-electromechanical systems (NEMS/MEMS). The FG micro- and nanosystems can be applied in different fields [1–5].

Dealing with the mechanical behavior of small-scale materials, neglecting the size-dependency might lead to producing inefficient MEMS or MEMS. Since the classical continuum mechanics is incapable of considering the size effect, several attempts have been made to develop different size-dependent elasticity theories. In this regard, one can mention: couple stress elasticity, nonlocal elasticity, surface stress elasticity and the strain gradient elasticity [6–11]. These theories and their modified forms have been frequently employed in studying small-scale structures [12–17]. Mindlin and Tiersten [6] and Toupin [18] proposed the classical couple stress theory which includes two classical and two additional material constants for isotropic elastic materials. After that, Yang et al. [19] represented the modified form of the couple stress theory (MCST). They considered the size effect using only one additional material length scale parameter besides two classical material constants.

MCST has been successfully employed in studying the size-dependent mechanical behavior of microstructures, especially microplates. In this direction, employing MCST, the size-dependent static and dynamic responses of Kirchhoff microplates were investigated in several papers [20–22]. Asghari [23] derived the governing equations of motion and boundary conditions for the geometrically nonlinear microplates with arbitrary shapes based on the MCST. Based on the Mindlin plate theory and MCST, Ke et al. [24] developed a size-dependent microplate model to investigate the free vibration behavior of microplates. Chen et al. [25] developed the Reddy microplate model for the bending analysis of composite laminated microplates. In particular, Ma et al. [26] developed the Mindlin microplate model, which was employed to examine the free vibration of microplates. It was observed that the natural frequency predicted by the size-dependent microplate model is higher than that predicted by the classical model, especially for thin plates. Wang et al. [27] proposed a non-classical mathematical model and an algorithm for the axisymmetrically nonlinear free vibration analysis of a circular microplate.

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Recently, MCST has been further developed to accommodate to size-dependent non-homogeneous microbeams and microplate models as well. The static and dynamic behavior of FGM Euler–Bernoulli and Timoshenko microbeams were studied by Asghari et al. [28,29], Reddy [30], Salamat-talab et al. [31], and Nateghi [32]. Reddy and Kim [33] developed a general nonlinear size-dependent third-order plate theory including geometric nonlinearity, and a gradient of the material properties. Reddy and Berry [34], based on MCST, power-law variation of the material, temperature-dependent properties, and the von Kármán geometric nonlinearity presented a microstructure-dependent nonlinear theory for the axisymmetric bending of circular microplates. Ke et al. [35] developed a non-classical microplate model and studied the free vibration behavior of annular FG microplates based on MCST, Mindlin plate theory and von Kármán geometric nonlinearity.

According to the literature, investigations regarding to the mechanical behavior of FG microplates are not well-developed and need further attention. Current paper is aimed to study the size-dependent free vibration behavior of FG rectangular Mindlin microplates including geometrical nonlinearity. The FG microplate is made of a mixture of metals and ceramics; a power law function is used to express the volume fraction of components. The MCST, Mindlin plate theory and the Hamilton's principle are employed to derive the governing equations of motion and associated boundary conditions. The set of nonlinear partial differential equations are discretized by the GDQ method and then a numerical Galerkin procedure is used with the aim of reducing the governing partial differential equations into a set of ordinary differential equations of Duffing-type. Since the vibration response of the microplate is periodic type, by using derivatives of a periodic base function, a set of periodic time differential matrix operators is introduced to discretize the Duffing equations on time domain. Finally, the pseudo arc-length continuation method is employed to find the large amplitude vibration response of the FG microplates. The effects of the material gradient index, length-to-thickness ratio, length scale parameter, and boundary conditions on the free vibration behavior of rectangular microplate are investigated.

2. Governing equations of size-dependent microplate

According to MCST, the strain energy of a continuum elastic medium occupying region V is defined by a function of strain tensor and gradient of the rotation vector as

$$U_m = \frac{1}{2}\int_V (\boldsymbol{\sigma} : \boldsymbol{\varepsilon} + \mathbf{m} : \boldsymbol{\chi}) dV, \tag{1}$$

where $\boldsymbol{\sigma}$,$\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$ are the Cauchy stress, Green strain and symmetric rotation gradient tensors, and $\mathbf{m}$ denotes the deviatoric part of couple stress tensor which for a linear isotropic elastic material can be defined as

$$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}), \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{2a}$$

$$\boldsymbol{\chi} = \frac{1}{2}(\nabla\boldsymbol{\theta} + (\nabla\boldsymbol{\theta})^{\mathrm{T}}), \quad \chi^s_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}), \quad \theta_i = \frac{1}{2}(curl(\mathbf{u}))_i, \tag{2b}$$

$$\boldsymbol{\sigma} = \lambda tr(\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon}, \quad \mathbf{m} = 2\mu l^2 \boldsymbol{\chi}, \tag{2c}$$

where $\mathbf{u}$ and $\boldsymbol{\theta}$ are the displacement and rotation vectors; λ and μ are Lame's constants defined as $\lambda = \frac{Ev}{(1+v)(1-2v)}$ and $\mu = \frac{E}{2(1+v)}$; in which v and E are Poisson's ratio and Young's modulus, respectively; and, l denotes a material length scale parameter, indicating the effect of couple stress.

A schematic of an FG Mindlin microplate with the length a, width b and thickness h, defined in the rectangular coordinate system $(0 \leqslant x \leqslant a,\ 0 \leqslant y \leqslant b, -h/2 \leqslant z \leqslant h/2)$ is illustrated in Fig. 1. The FG microplate is considered to be made of a mixture of ceramic and metal. Also, the bottom surface ($z = -h/2$) and top surface ($z = h/2$) of the microplate are metal-rich and ceramic-rich, respectively. Effective Young's modulus E, Poisson's ratio v and density ρ of FG microplate can be calculated by

$$E(z) = E_c V_c + E_m V_m, \quad v(z) = v_c V_c + v_m V_m, \\ \rho(z) = \rho_c V_c + \rho_m V_m, \tag{3}$$

where the subscripts m and c signal the metal and ceramic phases, respectively; V stands for the volume fraction of the phase materials determined via the power law function as [36]

$$V_c(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^k, \quad V_m(z) = 1 - \left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{4}$$

where k denotes the volume fraction exponent.

Based on the first-order shear deformation plate theory, the in-plane displacements can be stated as linear functions of the plate thickness and the transverse deflection is considered unchanged through the plate thickness; with regards to this theory, the displacement field in a Mindlin microplate can be described as

$$u_x = u(t,x,y) - z\psi_x(t,x,y), \quad u_y = v(t,x,y) - z\psi_y(t,x,y), \quad u_z = w(t,x,y), \tag{5}$$

in which $u(t,x,y)$ and $v(t,x,y)$ are mid-plane displacements, $w(t,x,y)$ is the lateral deflection of the microplate and (ψ_x,ψ_y) denote the rotations of the transverse normal about y- and x- axis, respectively. Also, t is the time. Inserting Eq. (5) into (2a) gives the nonzero components of the strain–displacement relations as follows:

$$\varepsilon_{xx} = \phi_0 - z\phi_1, \quad \varepsilon_{yy} = \varphi_0 - z\varphi_1, \quad \varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}(\kappa_0 - z\kappa_1), \\ \varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2}\gamma_1, \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\gamma_2, \tag{6}$$

where

$$\phi_0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \phi_1 = \frac{\partial \psi_x}{\partial x}, \quad \varphi_0 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \varphi_1 = \frac{\partial \psi_y}{\partial y}, \\ \kappa_0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \quad \kappa_1 = \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}, \quad \gamma_1 = \frac{\partial w}{\partial x} - \psi_x, \quad \gamma_2 = \frac{\partial w}{\partial y} - \psi_y. \tag{7}$$

Using Eqs. (5), (6) and (2) leads to the nonzero components of $\boldsymbol{\theta}$ and $\boldsymbol{\chi}$ as

$$\theta_x = \omega_x^0, \quad \theta_y = -\omega_y^0, \quad \theta_z = \omega_z^0 - z\omega_z^1, \tag{8}$$

where

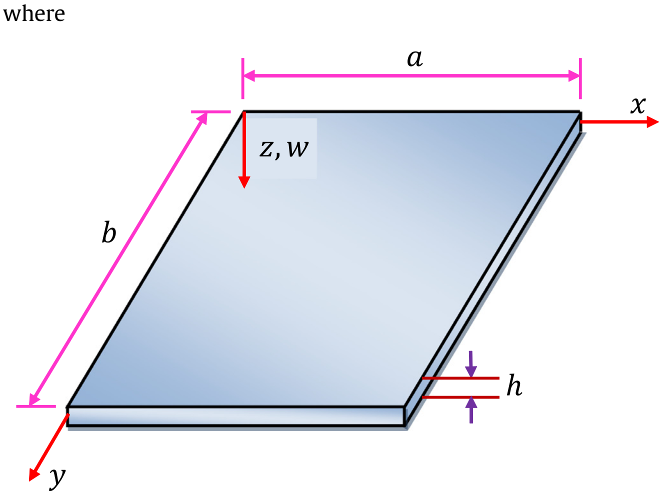

Fig. 1. Schematic of a microplate: kinematic parameters, coordinate system, geometry and loading.