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A contact extended isogeometric layerwise approach for the buckling analysis of delaminated composites

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ABSTRACT

Buckling of composite laminates with delaminations is studied based on a proposed isogeometric layerwise approach. The isogeometric approach employs the NURBS basis functions of the geometry's description to approximate the physical response in an isoparametric sense. Layerwise theories provide an accurate prediction of the three-dimensional structural responses of composite laminates while maintaining a two-dimensional data structure. Usually, to model the delaminations through thickness, the displacement field is enriched with a unit step Heaviside function which allows for discontinuities at the delamination interfaces. An improved implementation of a displacement-based layerwise theory based on the isogeometric paradigm is proposed. The delaminated and undelaminated regions of the laminate are modeled as separate patches. For the undelaminated and delaminated patches respectively, the $\mathcal{C}^0$-continuity and discontinuity conditions of the displacement field at the ply interfaces can be easily facilitated within the isogeometric framework. More reliable and accurate buckling loads are obtained by considering a contact analysis across the delamination interfaces to avoid physically inadmissible buckling modes. The proposed model is verified using laminated composite beam plates. The results are compared to a classical layerwise approach. The numerical results confirm the accuracy of the proposed isogeometric model.

1. Introduction

With the increasing use of composite laminates in modern industries, especially in the aerospace industry, various methods have been proposed for the analysis of laminated composite structures in a variety of areas, such as linear buckling [1,2], non-linear buckling [3,4], wave propagation [5,6] or stress analysis [7,8]. A broad overview of laminated composite materials and the related mathematical models can be found e.g. in Carrera [9] or Reddy [10]. Depending on the displacement or stress expansions through the laminate thickness, two major analysis strategies are distinguished: equivalent single layer methods (ESL) and layerwise methods (LW). As is well known, ESL methods, i.e., classical or shear deformation laminate theory, often fail to accurately capture the complete three-dimensional stress field at the ply level in moderately thick and thick laminates due to the fact that transverse strain components are incorrectly assumed to be continuous across the interface of dissimilar materials which entails a non-physical local discontinuity of the transverse stress components. In Demasi's paper [11], a special ESL method called the 'Variational Asymptotic Plate and Shell Analysis' (VAPAS) is developed. It is reported that, except for extreme cases of thick sandwich structures with very large modulus contrast, VAPAS can be used as an effective alternative to avoid expensive 3D finite element analysis. In contrast to ESL methods, displacement-based layerwise techniques assume separate displacement field expansions in in-plane and out-of-plane directions. Following equilibrium considerations, the transverse displacement component is defined to be $\mathcal{C}^0$-continuous at ply interfaces and thereby yield a more accurate description of the complete stress state.

In Robbins's [12] and Reddy's [13] work, one-dimensional Lagrangian finite elements are used through the thickness to approximate three components of the displacement field which automatically enforces $\mathcal{C}^0$-continuity conditions at layer interfaces. This approach results in a continuous in-plane and discontinuous transverse strain fields, hence, allowing for the possibility of continuous transverse stresses at the layer interfaces. Compared to conventional 3-D finite element models, Reddy's layerwise model is computationally more efficient while retaining the same modelling capabilities.

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Delamination in composite laminates is a common defect either pre-existing or generated during their service times by impact, fatigue etc. The presence of delamination may cause an obvious reduction of the load carrying capacity of a laminate. For example, due to the local instability in the vicinity of a delamination, the laminate may buckle at a level of compressive loads well below the design value for the undelaminated composite structure.

Generally, there are two ways of dealing with buckling analysis. The first one follows the history of load–displacement variations in a non-linear analysis [14,4], the second is based on the solution of an eigenvalue problem [15] to obtain the buckling loads and mode shapes. A one-dimensional analytical model was first proposed by Chai et al. [16] to investigate the buckling behavior of a delaminated composite structure. Following Simitses et al. [17,18], proposing a one-dimensional beam-plate theory to predict the buckling loads and growth of delamination, the influences of both delamination geometries and positions of the buckling loads are studied. Kardomateas and Schmueser [19] and Chen [20] include the shear deformation effect in the one-dimensional model which reduces the overestimation of the buckling load for a delaminated composite structure.

Apart from the above one-dimensional models, Barbero [21] proposes a layerwise plate theory to model the delamination in composite laminates, in which, the displacement field is enriched with a unit step function to allow for separation and slipping at delamination surfaces. Lee et al. [22] follows this layerwise approach to study the buckling problem of axially loaded composite beam plates based on the solution of an eigenvalue problem. The proposed layerwise approach yields accurate results at a reduced computing time. Most of the references above which address the buckling of single or multiple delaminations of laminated composites, e.g. [22], do not consider buckling modes where contact between delaminated plies occurs. In general, physically inadmissible mode shapes with overlapping plies may appear, hence, it is necessary to include a contact treatment in the buckling analysis of delaminated laminates. Peck and Springer [23] include a contact model in the eigenvalue buckling analysis with reasonable results. Their contact model considers a delaminated ply resting on an elastic foundation. Regarding the contact problem, Suemasu [24] adds imaginary springs between two delaminated layers to provide the resistance forces and moments eliminating the inadmissible mode shapes. The same idea is adopted in Sekine's [25] and Kouchakzadeh's [15] work. A similar artificial spring approach based on contact forces is presented in Hu [26] for Reissner–Mindlin plates. The contact forces of that approach are found from a sensitivity-based update iteration scheme.

Isogeometric analysis, recently introduced by Hughes et al. [27], is a novel concept in computational mechanics aimed at unifying computer aided design (CAD) and finite element analysis (FEA). In contrast to classical FEA where both the geometry and the unknown solution field is approximated with Lagrange polynomials, isogeometric analysis (IGA) employs the basis functions used to describe the geometry to approximate the physical response in an isoparametric sense. Moreover, the unique k-refinement of isogeometric analysis enables us to control the continuity of the basis [28], and may facilitate the layerwise modeling of composite laminates which needs $\mathcal{C}^0$-continuity of the displacement field at layer interfaces. The accuracy and efficiency of the isogeometric paradigm have been demonstrated by a number of researchers in the past years, see e.g. [29–38]. For the analysis of laminated composite structures following the isogeometric concept to date significantly less publications can be found, e.g. [39–45,37,38,46]. Most of these works are based on the classical lamination theory (CLT) or the first-order shear deformation theory both belonging to the class of ESL methods. Recently, Guo et al. [38] have proposed an isogeometric layerwise theory in which a NURBS basis is used to interpolate the in-plane and out-of-plane displacements independently, and the $\mathcal{C}^0$-continuity at layer interfaces can be ensured naturally by conscious use of refinement schemes of isogeometric analysis.

In the research reported in this paper we employed the higher order and higher continuity properties of NURBS in the framework of isogeometric buckling analyses of delaminated composite structures with a clear focus on the through-thickness buckling behaviour. Despite focussing on B-spline models to demonstrate the performance of the introduced method we provided a complete description of the basic principles of isogeometric analysis in terms of NURBS modelling and kept this terminology throughout the paper. We have demonstrated the reliability and accuracy of a contact analysis extension of the IGA framework to eliminate physically inadmissible buckling states which may result from overlapping plies. We exploited the smoothness of the buckled NURBS geometry to eliminate convergence problems arising from non-smooth surface discretizations of traditional finite element based contact analysis. Although, in recent years a number of surface smoothing techniques have been proposed for finite element based contact models [47,48], they inevitably introduce an additional geometry and may cause additional computational effort [49]. The advantageous use of NURBS-based isogeometric contact analysis has been demonstrated previously in [50,51,49]. We further exploited the unique refinement schemes of isogeometric analysis to control the continuity properties over the ply stacking and the laminate extension plane that allows h-refinement and simple order elevation while keeping the necessary $\mathcal{C}^0$ continuity at the ply interface in the applied layerwise approach. In Section 2 we give a brief summary about the layerwise lamina theory in the framework of isogeometric analysis. The set of equations governing the linear buckling analysis are provided in Section 3. We further present in this section the applied modeling approach including the coupling of multiple patches for an appropriate assembly of the delamination model. In Section 4 we introduce the contact kinematics of our method and a surface-to-surface contact element formulation. We provide a conceptual description of the contact analysis and show the complete buckling analysis procedure in algorithmic form. Various examples addressing accuracy, reliability and convergence behavior of the analysis are presented in Section 5. We systematically analyse the influence of the location and the size of single and multiple delaminations. The proposed contact enriched isogeometric buckling analysis approach is summarized and conclusions given in Section 6.

2. Isogeometric layerwise theory

We start with a brief summary of the isogeometric analysis focusing on the use of B-spline and NURBS as appropriate basis functions in composite laminate modeling. We further summarize the basic concept of the layerwise theory pursued in this contribution and provide the set of governing equations which is used in the following to formulate the variational formulation of the buckling analysis.

2.1. B-spline & NURBS basis functions

Isogeometric analysis follows the principles of the isoparametric paradigm, as known from classical finite elements, based on B-splines or non-uniform rational B-splines (NURBS) which are the basis of todays CAD tools [52,53]. Using the same basis functions for the interpolation of the geometric and the physical model aims for a significant reduction of the modeling effort in numerical simulation of CAD-based models while profiting from the smoothness, continuity and adaptivity properties of NURBS [27].