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Size-dependent analysis of a three-layer microbeam including electromechanical coupling

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ABSTRACT

Strain gradients can induce polarization even in centrosymmetric materials. A size-dependent model of a three-layer microbeam including a flexoelectric dielectric layer is proposed based on the theory presented by Hadjesfandiari. The governing equations, initial conditions and boundary conditions are derived by utilizing the Hamilton's principle. Both the static bending and free vibration problems of cantilever and simply supported microbeams are solved. Numerical results reveal that for both cantilever beam and simply supported beam, the absolute values of voltages induced in the piezoelectric process and deflections generated in the inverse piezoelectric process decrease as the characteristic size decreases and increase with the increase of flexoelectric coefficient. The first order natural frequency increases negligibly as the flexoelectric coefficient increases while decreases significantly with the increase of the characteristic size. The first order natural frequency shows obvious size effect, but the size effect is almost diminishing as the thickness of the beam is far greater than the material length scale parameter.

1. Introduction

Microbeams, as sensors, resonators and actuators, have been found widely applications in microelectromechanical system (MEMS). By coupling the mechanical and electrical fields, piezoelectric microbeams have attracted much attention in recent years [1–5]. According to their designed functions and sensitivities, the usual piezoelectric structures are unimorph and bimorph ones, having one or two piezoelectric layers, respectively [6]. In general, the piezoelectric microbeams are multi-layered structures including substrate, electrodes, piezoelectric layers, even elastic layers [7]. Up to now, many efforts have been devoted to the analysis of electromechanical coupling in dielectric solids.

A piezoelectric multilayered cantilever model considering the buffer layer and electrodes is established to evaluate the dynamic performance of the piezoelectric actuator by Peng et al. [8] based on the Bernoulli–Euler beam theory. Yang et al. [9] studies the interfacial mechanical behavior of laminate beams which consist of two piezoelectric facial sheets and an elastic core. The tip-deflection of a piezoelectric bimorph cantilever is analyzed by Huang et al. [10] in its static state. Moreover, for functionally graded piezoelectric beam, the static, free vibration and dynamic response are researched by Legy-Nazargah et al. [11] utilizing a finite element model. Alibeigloo [12] presents an analytical solution for functionally graded beam integrated with piezoelectric layers under an applied electric field and thermo-mechanical load.

All above works are based on the classical piezoelectric theory, where the relation between electric polarization and strain is described in non-centrosymmetric dielectrics. Although piezoelectricity is inherent only in non-centrosymmetric materials, a piezoelectric response can also be found in centrosymmetric materials [13]. An inhomogeneous strain field or the presence of the strain gradients can locally break inversion symmetry and may induce polarization in centrosymmetric dielectrics [14–17]. The coupling of strain gradient to polarization is expected to show a strong size dependence, which is known as flexoelectric effect in some circles. Fousek et al. [18] are the first to show fabrication of piezoelectric composites using piezoelectric materials is not a necessary condition. Cross even has developed a piezoelectric composite containing no piezoelectric element [19]. The linear electromechanical coupling in isotropic materials and size-effect phenomena in piezoelectric solids have been reported in some experiments [20–23]. Due to neglecting the microstructure, the classical piezoelectric theory fails to capture the size dependence of piezoelectric solids. Thus, the size-dependent piezoelectric theories have been proposed to account for the size effect.

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The size-dependent piezoelectric theories are developed from the strain gradient theories which achieve giant success in materials without electromechanical coupling. According to the deformation metrics used, the strain gradient theories are classified into couple stress theories and general strain gradient theories. The couple stress theory [24–25] uses the rotation gradients as the high-order deformation metrics and it includes two high-order material constants. The general strain gradient theory [26] uses the second-order deformation gradients (the first-order strain gradients) as the high-order deformation metrics and it has five higher-order material constants. The rotation gradients are the skew-symmetric part of the second-order deformation gradients. The general strain gradient theory, hence, can reduce to the couple stress theory. In view of the practicability, the strain gradient theories are simplified by introducing a higher-order equilibrium condition. Through neglecting the skew-symmetric part of the rotation gradients, Yang et al. [27] develop the modified couple stress theory with only one high-order material constant and Lam et al. [28] propose the modified strain gradient elasticity theory including three high-order material constants. Recently, by considering true continuum kinematical displacement and rotation, Hajesfandiari and Dargush [29] demonstrate the couple tensor is skew-symmetric. Thus, they present the consistent couple stress theory by adopting the skew-symmetric part of the rotation gradients as the curvature tensor.

From the strain gradient theories, the high-order theories containing electromechanical coupling effect are developed. A piezoelectric theory with rotation gradient effect is proposed by Wang et al. [30] in the framework of the couple stress theory. Hu and Shen [31] establish a theory concerning the strain/electric field gradient with the surface and electrostatic force effect. However, as stated previously, these theories suffer from its dependence on inaccuracy enough strain gradient theories. Hadjesfandiari [32], hence, develops a piezoelectric couple stress theory in which the size-dependent piezoelectric effect is related to the skew-symmetric part of the rotation gradient.

By applying the high-order piezoelectric theories, the size-dependent electromechanical coupling effects are researched. Liang and Shen [33] develop a size-dependent Bernoulli–Euler beam model for the piezoelectric nanowires based on the theory proposed by Hu and Shen. The first-order strain gradient effect in micro piezoelectric bimorph is investigated by Hu et al. [34]. In addition, the nonlinear vibration of the piezoelectric nanobeams is analyzed by Ke et al. [35] based on the nonlocal theory and Timoshenko beam theory.

This paper represents a further research of Hadjesfandiari's work that establishes a piezoelectric couple stress elasticity theory predicting the size effect and electromechanical coupling effect reported in some isotropic materials. A size-dependent three-layer microbeam model containing substrate, flexoelectric layer and electrode is developed. The governing equations, initial conditions and boundary conditions are derived by using the Hamilton's principle. Two boundary value problems (one for cantilever beam and another for simply supported beam) are assessed and the size dependences of the electromechanical coupling effect are discussed.

2. Size-dependent three-layer microbeam model including electromechanical coupling

2.1. Size-dependent piezoelectricity

In order to account for the size effect phenomena of piezoelectric solids and linear electromechanical coupling in isotropic materials, Hadjesfandiari [32] develops a consistent size-dependent piezoelectric theory. This theory is based on the couple stress theory, in which the skew-symmetrical part of the macroscopic rotation gradient is treated as the measure deformation. The electromechanical enthalpy is expressed as,

$$H = \frac{1}{2}\lambda\varepsilon_{ii}\varepsilon_{jj} + \mu\varepsilon_{ij}\varepsilon_{ij} + 8\mu l^2\kappa_i\kappa_i - \frac{1}{2}\varepsilon E_i E_i - 4fE_i\kappa_i, \tag{1}$$

where λ and μ are the Lame constants, l is the independent material length scale parameter, ε is the permittivity, f is the flexoelectric coefficient, ε_{ij} is the strain tensor, κ_i is the mean curvature vector, and E_i is the electric field vector. These deformation measures are defined as

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{2}$$

$$\kappa_i = \frac{1}{4}(u_{j,ij} - u_{i,jj}), \tag{3}$$

$$E_i = -\phi_{,i}, \tag{4}$$

in which u_i is the displacement vector and φ is the electric potential.

2.2. Governing equations, initial conditions and boundary conditions

As shown in Fig. 1, an isotropic homogeneous flexoelectric dielectric is bonded to an elastic substrate and an electrode is laid on the top surface of flexoelectric layer. The width of microbeam is b and the surface heights of three layers denote h_1, h_2 and h_3, respectively. The elastic three-layer microbeam is subjected to a lateral load $q(x,t)$ along its length L and a voltage V_0 between the upper and lower surfaces of the flexoelectric layer. A Cartesian coordinate system is adopted in this model, where x-axis is established at the subface of beam. For the Bernoulli–Euler beam, the displacement components [36] can be taken as

$$u = u_0(x,t) - z\frac{\partial w(x,t)}{\partial x}, \quad v = 0, \quad w = w(x,t), \tag{5}$$

where u, v, w, are the x-, y-, z-components of the displacement vector, u_0 is the axial displacement on the x–y plane and t is time. For a slender beam with a large aspect ratio, the electric field E_x can be neglected and the only relevant electric component is E_z [37,38]. Hence, the electric potential can be assumed to be

$$\phi = \phi(z,t). \tag{6}$$

From the displacement field and electric potential, the strain, curvature and electric field components can be calculated by substituting Eqs. (5) and (6)into geometric equations, Eqs. (2)–(4). The non-zero components are written as

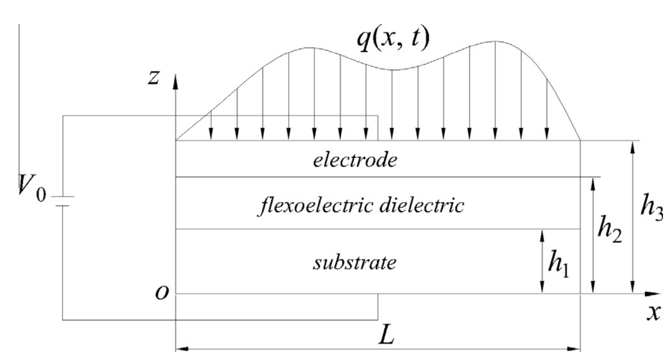

Fig. 1. Longitudinal section of a three-layer microbeam and its coordinate system.