Coupling of heterogeneous kinematics and Finite Element approximations applied to composite beam structures

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ABSTRACT

In the framework of the modeling of composite beam structures, the eXtended Variational Formulation (XVF) is carried out to couple different kinematics. The purpose is to take advantages of efficient models and reduce the overall computational cost without loss of local precision. In this way, the structure is divided into non-overlapping domains with different kinematics. Local domains of interest are described with advanced models, such as refined Sinus model, to precisely describe local behavior, while the remaining global domain uses simple classical models. Each of the kinematics needs a suitable Finite Element (FE) approximation, therefore the coupling of different FE approximations is also addressed. The present approach is assessed on homogeneous and sandwich structures. It is compared with classical Multiple Point Constraints vias penalty. The study has shown the need of the introduction of a new operator for the layered structures. The results are very promising.

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1. Introduction

Composite structures are widely used in industry due to their excellent weight-specific mechanical properties. However, the price to be paid is a more complex behavior with respect to classical monolithic ones. In particular, a high computational effort is required for obtaining accurate stress fields due to material heterogeneity and anisotropy, even if their geometry often calls for the application of structural models of reduced dimensionality like beams (slender solids) and plate/shells (thin solids). Nevertheless, in many cases high stress gradients are limited to several local domains of interest, while large portions of the structure can effectively profit of the global geometric slenderness and/or thinness. In light of an optimization of the computational effort, it appears thus interesting to model these large portions of the structure by means of classical structural models, which rely on low-order kinematics assumptions such as Euler–Bernoulli's or Kirchhoff–Love's, and to employ a refined description for the local regions in which an adequate resolution of the stress gradients is required. Such global–local analysis technique is well known, in which most often a solid (3D) model is used for the local domains [1]. Instead of using a full 3D model, in many cases it is however possible to employ structural models of lower dimensionality and refined, higher-order kinematics. This way, a further reduction of the computational

effort may be achieved without degrading the local accuracy in the framework of a global–local analysis method. Within this scope, the present work aims at developing a dedicated technique to couple beam Finite Elements formulated with heterogeneous kinematics, where a low-order classical kinematics is used in a global, "simple" sub-domain and a refined, higher-order one in a local, "complex" sub-domain.

First applied to the multiphysics environment, the coupling of different sub-domains has been a subject of several research activities from which different techniques are available today. These can take into account the coupling of domains of homogeneous dimensionality but heterogeneous kinematical description, or domains of heterogeneous dimensionality and homogeneous kinematical description. In the following, the commonly used techniques shall be classified following the topology of the arrangement of the domains: sub-domains with complete and partial overlap as well as non-overlapping sub-domains will be distinguished. Such a classification appears in fact more intuitive than a comparison including the question of homogeneous or heterogeneous dimensionality.

The complete overlap is first considered. Fish et al. [2,3] improved the accuracy of results by superimposing additional mesh of higher-order hierarchical elements in the region of interest. This method is called the superposition version of the Finite Element Method (FEM), also known as the s-Version of the FEM. Homogeneous boundary conditions along the common boundary are applied to keep the C^0 continuity of the displacements. The displacement field in the local area is thus obtained as the sum of the

higher order mesh and the global mesh. The solution for homogeneous dimensional problems is obtained directly with incompatible meshes. Reddy and Robbins [4] proposed to combine the variable kinematics Finite Elements with the s-Version approach. This represents an extension of Fish's method in that it couples Finite Elements with different mathematical model types. So, the local overlay mesh of variable kinematic elements can include both Equivalent Single Layer and LayerWise approaches. While both mentioned techniques resort to a simultaneous solution of the overlapping domains, Gendre et al. [5] developed an iterative method for domains with different description. Their two-scale analysis method for local sub-domains combines local and global contributions. The global domain is assumed to have linear elastic properties, which can be also homogenized, while material and geometric non-linearity can be included for the local domain. It uses a two-scale approximation of the Schur complement of the local domain's stiffness matrix. The use of a weighted combination of Dirichlet and Neumann boundary conditions on the local domain enables relatively low number of iterations and assures the convergence of the solution.

As far as the techniques with partial overlap are concerned, Ben Dhia et al. [6] proposed the so-called Arlequin method. Three regions are here considered: a domain using a simple kinematical model, a domain using a complex kinematical model and the sub-domain where both domains overlap. The connection in the superposition zone is performed in a weak sense via Lagrange Multipliers, and the total internal energy is partitioned between the overlapping models. The Arlequin method was systematically used by Hu and coworkers to analyze sandwich structures for both linear [7] and nonlinear applications [8,9]. In these applications, the Arlequin framework has been used to couple coarse and refined models, including both dimensionally homogeneous and heterogeneous models, as well as a FE model with a known solution. Biscani and coworkers continued this approach by referring to dimensionally homogeneous sub-domains with low-order and high-order kinematics for beams [10] and plates [11]. Note that the superposition volume of the Arlequin method can be degenerated to a surface coupling of the domains. However, some perturbations are induced close to the interface due to the coupling formulation between the domains.

Finally, non-overlapping techniques are widely used in open literature. The classical Lagrange Multipliers method allows to link the displacements between two adjacent subdomains by adding an interface constraint functional in the mechanical formulation of the problem. The interface potential was first proposed by Prager [12] to treat internal physical discontinuities by linking the unknowns of the sub-domains through a single Lagrange Multipliers field. This classical, two-fields formulation was used in [13] to combine variable kinematic models in the framework of the Carrera's Unified Formulation for beam structures. The mortar method is also mentioned, which provides the well-suited Lagrange Multipliers space [14]. The Lagrange Multipliers method was also employed for a global/local approach with incompatible FE meshes by introducing an independent displacement field at the interface (three-fields formulation) [15]. Ransom extended this technique for different solution methods, like finite differences, Finite Elements and finite volumes [16]. Note the substantial works of Park and Felippa and their systematic development of hybrid functionals for the analysis of partitioned systems with Lagrange Multipliers [17,18]. In these works, the rigid-body modes in the governing equation of floating subdomains are explicitly separated in order to attain the solvability condition. Both static and dynamic problems are addressed. However, these approaches require a degree of freedom compatibility.

As an alternative, Blanco et al. [19] developed the so-called eXtended Variational Formulation (XVF), in which two Lagrange Multipliers fields are introduced. It allows to couple structural models with different dimensionality involving different types of degrees of freedom. As Lagrange Multipliers can disturb the banded form of the linear system describing the mechanical problem, Kim [20] developed a special interface element, avoiding Lagrange Multipliers. The continuity of the displacements is ensured through the interface element domain with new specific shape functions constructing by moving least square approximations. The numerical integration is a difficult task in this approach.

In commercial FE codes, global/local analysis can be performed. The overall domain is described with a simple or low order kinematical model, and local sub-domains can be modeled by refined descriptions. At the common border, the results of the simple model are used as boundary conditions of the local sub-domain. Calculations are usually carried out in at least two subsequently steps, but iterative methods can also be applied. We can also mention Multiple Point Constraints (MPC) which are available in commercial codes. A kinematics constraint on the degrees of freedom (dofs) can be imposed at the common interface between two models. This technique does not require additional dofs. Nevertheless, it does not rely on an energy principle. Recently, a MPC formulation for eigenfrequency calculation and dynamic contact problems was developed by Hetherington et al. [21].

In the present work, the so-called XVF approach is extended to couple heterogeneous kinematics for the modeling of composite structures within a non-overlapping scheme. The local region of interest is modeled with a refined Sinus approach previously introduced in [22], which has shown very interesting features, while classical simple models can be used for the "global" portion of the structure. A compatibility condition is imposed at the common interface by introducing two Lagrange Multipliers fields that are to be chosen according to the specific kinematics used in the adjacent sub-domains. No additional unknown has to be introduced. The standard MPC approach via penalty method will be addressed for the sake of comparison.

We now outline the remainder of the article. The formulation of the XVF approach is first given. The classical models (namely Euler Bernoulli and Timoshenko) as well as the refined Sinus model are subsequently described. Note that all these models have very different features. The transverse shear stress deduced from these approaches are rather different. Moreover, only the refined model takes into account the transverse normal deformation. We finally focus on the coupling of these different models within the particular XVF framework and on the resulting FEM formulation. Numerical assessments are provided on homogeneous and sandwich beams for both XVF and penalty methods. A preliminary study is devoted to the evaluation of each model without coupling. The influence of the position and the size of the complex zones are discussed. Special attention is also dedicated to the influence of this coupling on the results close to the interface between the domains.

2. eXtended Variational Formulation (XVF)

Let us consider an elastic structure defined in a domain $\Omega \subset \mathcal{R}^3$ bounded by $\Gamma = \Gamma_D \cup \Gamma_N$ with $\Gamma_D \cap \Gamma_N = \emptyset$. The structure is submitted to prescribed body forces $\boldsymbol{f}$ on Ω and surface forces $\boldsymbol{t}$ applied on Γ_N. The displacement $\bar{\boldsymbol{u}}$ is imposed on Γ_D. In the present scope, a beam structure occupying the domain $\Omega = [0, l] \times S$ is addressed, where S is the constant cross-section of the beam, and l is the length. Using the variational formulation, one can state:

Find $\boldsymbol{u} \in \mathcal{U}$ such that:

$$\int_\Omega \boldsymbol{\sigma}(\boldsymbol{u})\,\delta\boldsymbol{\epsilon}(\boldsymbol{u})\,d\Omega = \int_\Omega \boldsymbol{f} \cdot \delta\boldsymbol{u}\,d\Omega + \int_{\Gamma_N} \boldsymbol{t} \cdot \delta\boldsymbol{u}\,d\partial\Omega \quad \forall \delta\boldsymbol{u} \in \delta\mathcal{U} \tag{1}$$

where