Composite Structures 116 (2014) 336–345

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Guided wave in multilayered piezoelectric–piezomagnetic bars with rectangular cross-sections

J.G. Yu [a,b,*], J.E. Lefebvre [c,d,e], Ch. Zhang [b]

[a] School of Mechanical and Power Engineering, Henan Polytechnic University, Jiaozuo 454003, PR China
[b] Department of Civil Engineering, University of Siegen, D-57068 Siegen, Germany
[c] Univ Lille Nord de France, F-59000 Lille, France
[d] UVHC, IEMN-DOAE, F-59313 Valenciennes Cedex 9, France
[e] CNRS, UMR 8520, F-59650 Villeneuve d'Ascq, France

ARTICLE INFO

Article history:
Available online 10 May 2014

Keywords:
Wave propagation
Rectangular bar
Piezoelectric–piezomagnetic material
Double orthogonal polynomials
Dispersion curves
Mechanical displacement profiles

ABSTRACT

For the purpose of design and optimization of multilayered piezoelectric–piezomagnetic material (PPC) transducers, wave propagation in these structures has received much attention in past ten years. But the research objects of previous works are only for semi-infinite structures and one-dimensional structures, i.e., structures with a finite dimension in only one direction, such as horizontally infinite flat plates and axially infinite hollow cylinders. This paper proposes a double orthogonal polynomial series approach to solve the wave propagation problem in a two-dimensional (2-D) structure, namely, a layered PPC bar with a rectangular cross-section. Through numerical comparison with the available reference results for a purely elastic layered rectangular bar, the validity of the double polynomial approach is illustrated. The dispersion curves and mechanical displacement profiles of various layered PPC rectangular bars are calculated to reveal the wave propagation characteristics.

1. Introduction

Over the past ten years, piezoelectric–piezomagnetic composites (PPC) have received considerable research effort with their increasing usage in various applications including sensors, actuators and storage devices [1–4]. For the purpose of design and optimization of PPC transducers, wave propagation in various PPC attracted many researchers.

Chen and Shen [5] obtained effective wave velocity and attenuation factor when axial shear magneto-electro-elastic waves propagate in piezoelectric–piezomagnetic composites. By using the state space approach, Zhou et al. [6] investigated the bulk wave propagation in laminated piezomagnetic piezoelectric plates with initial stresses and imperfect interface. Using the propagator matrix and state-vector (or state space) approaches, an analytical treatment is presented for the propagation of harmonic waves in magneto-electro-elastic multilayered plates by Chen et al. [7]. By using Legendre orthogonal polynomial series expansion approach, Yu et al. investigated the guided waves in imhomogeneous magneto-electro-elastic hollow cylinders [8] and spherical curved plates [9].

In order to analyze the band gaps, wave propagation in piezoelectric–piezomagnetic periodically layered structures received attentions [10–12]. Li et al. [13] discussed the penetration depth of the Bleustein–Gulyaev waves in a functionally graded transversely isotropic electro-magneto-elastic half-space. SH waves propagating in piezoelectric–piezomagnetic layered structures with imperfect interfaces were investigated by Sun et al. [14] and Nie et al. [15]. Pang and Liu [16] discussed the reflection and transmission of plane waves at an imperfectly bonded interface between piezoelectric–piezomagnetic media. By using the Jacobi elliptic function expansion method, Xue et al. [17] investigated the solitary waves in a magneto-electro-elastic circular bar. Matar et al. [18] used Legendre and Laguerre polynomial approach for modeling of wave propagation in layered magneto-electro-elastic media. Xue and Pan [19] discussed the influences of the gradient factor on the longitudinal wave along a functionally graded magneto-electro-elastic bar. Zhang et al. [20] studied the influences of initial stresses on Rayleigh wave propagation in a magneto-electro-elastic half-space.

As can be seen from the above simple review, wave motions in many magneto-electro-elastic structures have been considered. But these structures are almost only for semi-infinite structures

* Corresponding author at: School of Mechanical and Power Engineering, Henan Polytechnic University, Jiaozuo 454003, PR China. Tel.: +86 13693919651.
E-mail address: yu@emails.bjut.edu.cn (J.G. Yu).

http://dx.doi.org/10.1016/j.compstruct.2014.04.025
0263-8223/

and one-dimensional structures, i.e. structures having a finite dimension in only one direction, such as horizontally infinite flat plates and axially infinite hollow cylinders. In practical applications, many sensitive elements have limited finite dimension in two directions. One-dimensional models are thus not suitable for these structures. On the other hand, wave propagation in purely elastic 2-D structures has been addressed by researchers. Kastrzhitskaya and Meleshko [21] proposed an exact analytical method for an effective solution of the problem of rectangular waveguide. Taweel et al. [22] used a semi-analytical finite element method to study the layered rectangular bars. Also using a semi-analytical finite element method, Hayashi et al. [23] analyzed square bars and rail waveguides. Gunawan and Hirose [24] investigated the rectangular bars by boundary element method. By means of standard commercial finite element codes, Sorohan et al. [25] investigated a layered composite plate and a square tube.

In this paper, a double orthogonal polynomial series approach is proposed to solve wave propagation in a 2-D PPC structure, namely a multilayered PPC bar with a rectangular cross-section. Traction-free and open-circuit boundary conditions are assumed in this analysis. Two cases are considered: the material stacking direction and the polarizing direction are identical and orthogonal to each other, respectively. The dispersion curves and the mechanical displacement profiles of various layered PPC rectangular bars are presented and discussed.

2. Mathematics and formulation of the problem

We consider a multilayered piezoelectric rectangular bar which is infinite in the wave propagation direction. Its width is d, the total height is $h = h_N$, and the stacking direction is in the z-direction, as shown in Fig. 1. Its polarization direction is in the z direction. The origin of the Cartesian coordinate system is located at a corner of the rectangular cross-section and the bar lies in the positive $y-z$-region, where the cross-section is defined by the region $0 \leqslant z \leqslant h$ and $0 \leqslant y \leqslant d$.

For the wave propagation problem considered in this paper, the body forces, electric charges and current density are assumed to be zero. Thus, the dynamic equations for the rectangular bar are governed by

$$\begin{aligned}
&\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{xy}}{\partial y}+\frac{\partial T_{xz}}{\partial z}=\rho\frac{\partial^2 u_x}{\partial t^2}\\
&\frac{\partial T_{xy}}{\partial x}+\frac{\partial T_{yy}}{\partial y}+\frac{\partial T_{yz}}{\partial z}=\rho\frac{\partial^2 u_y}{\partial t^2}\\
&\frac{\partial T_{xz}}{\partial x}+\frac{\partial T_{yz}}{\partial y}+\frac{\partial T_{zz}}{\partial z}=\rho\frac{\partial^2 u_z}{\partial t^2}\\
&\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=0\\
&\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0
\end{aligned} \tag{1}$$

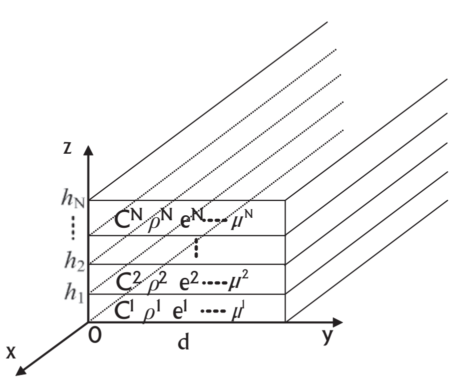

Fig. 1. Schematic diagram of a multilayered rectangular bar.

where T_{ij}, u_i, D_i and B_i are the stress, elastic displacement, electric displacement and magnetic induction components respectively and ρ is the density of the material.

The relationships between the generalized strain and generalized displacement components can be expressed as

$$\begin{aligned}
&\varepsilon_{xx}=\frac{\partial u_x}{\partial x},\ \varepsilon_{yy}=\frac{\partial u_y}{\partial y},\ \varepsilon_{zz}=\frac{\partial u_z}{\partial z},\ \varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right), e\\
&\varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right),\\
&\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right),\ E_x=-\frac{\partial\phi}{\partial x},\ E_y=-\frac{\partial\phi}{\partial y},\ E_z=-\frac{\partial\phi}{\partial z},\\
&H_x=-\frac{\partial\Psi}{\partial x},\ H_y=-\frac{\partial\Psi}{\partial y},\ H_z=-\frac{\partial\Psi}{\partial z}
\end{aligned} \tag{2}$$

where ε_{ij}, E_i and H_i are the strain, electric field and magnetic field, ϕ and Ψ the electric potential and the magnetic potential respectively.

We introduce the function $\mathrm{I}(y,z)$

$$\mathrm{I}(y,z)=\pi(y)\pi(z)=\begin{cases}1, & 0\leqslant y\leqslant d \text{ and } 0\leqslant z\leqslant h\\ 0, & \text{elsewhere}\end{cases}, \tag{3}$$

where $\pi(y)$ and $\pi(z)$ are rectangular window functions $\pi(y)=\begin{cases}1, & 0\leqslant y\leqslant \mathrm{d}\\ 0, & \text{elsewhere}\end{cases}$ and $\pi(z)=\begin{cases}1, & 0\leqslant z\leqslant h\\ 0, & \text{elsewhere}\end{cases}$. By introducing the function $\mathrm{I}(y,z)$, the traction free and open circuit boundary conditions $(T_{zz}=T_{xz}=T_{yz}=T_{yy}=T_{xy}=D_z=D_y=B_z=B_y=0$ at the four boundaries) are automatically incorporated in the constitutive relations of the bar (here the material is assumed to be orthotropic) [26]

$$\begin{aligned}
&T_{xx}=C_{11}\varepsilon_{xx}+C_{12}\varepsilon_{yy}+C_{13}\varepsilon_{zz}-e_{31}E_z-q_{31}H_z,\\
&T_{yy}=(C_{12}\varepsilon_{xx}+C_{22}\varepsilon_{yy}+C_{23}\varepsilon_{zz}-e_{32}E_z-q_{32}H_z)\mathrm{I}(y,z),\\
&T_{zz}=(C_{13}\varepsilon_{xx}+C_{23}\varepsilon_{yy}+C_{33}\varepsilon_{zz}-e_{33}E_z-q_{33}H_z)\mathrm{I}(y,z),\\
&T_{yz}=(2C_{44}\varepsilon_{yz}-e_{24}E_y-q_{24}H_y)\mathrm{I}(y,z),\\
&T_{xz}=(2C_{55}\varepsilon_{xz}-e_{15}E_x-q_{15}H_x)\mathrm{I}(y,z),\\
&T_{xy}=2C_{66}\varepsilon_{xy}\mathrm{I}(y,z),
\end{aligned} \tag{4a}$$

$$\begin{aligned}
&D_x=2e_{15}\varepsilon_{xz}+\in_{11}E_x+g_{11}H_x\\
&D_y=(2e_{24}\varepsilon_{yz}+\in_{22}E_y+g_{22}H_y)\mathrm{I}(y,z)\\
&D_z=(e_{31}\varepsilon_{xx}+e_{32}\varepsilon_{yy}+e_{33}\varepsilon_{zz}+\in_{33}E_z+g_{33}H_z)\mathrm{I}(y,z)
\end{aligned} \tag{4b}$$

$$\begin{aligned}
&B_x=2q_{15}\varepsilon_{xz}+g_{11}E_x+\mu_{11}H_x\\
&B_y=(2q_{24}\varepsilon_{yz}+g_{22}E_y+\mu_{22}H_y)\mathrm{I}(y,z)\\
&B_z=(q_{31}\varepsilon_{xx}+q_{32}\varepsilon_{yy}+q_{33}\varepsilon_{zz}+g_{33}E_z+\mu_{33}H_z)\mathrm{I}(y,z)
\end{aligned} \tag{4c}$$

where C_{ij}, e_{ij} and q_{ij} are the elastic, piezoelectric, and piezomagnetic coefficients respectively; $\in_{ij}$, g_{ij}, and μ_{ij} are the dielectric, magnetoelectric, and magnetic permeability coefficients respectively.

The layered bar with a stacking direction being in the z-direction is denoted as z-directional layered bar. The layered bars considered in the following are all z-directional layered bars if not indicated specifically. The elastic constants of the layered bar are expressed as

$$C_{ij}=\sum_{n=1}^{N}C_{ij}^{n}\pi_{h_{n-1},h_n}(z), \tag{5a}$$

where N is the number of the layers; C_{ij}^n is the elastic constant of the nth layer and $\pi_{h_{n-1},\,h_n}(z)$ is the rectangular window function. Similarly, other material coefficients can be expressed as

$$\begin{aligned}
&e_{ij}=\sum_{n=1}^{N}e_{ij}^{n}\pi_{h_{n-1},h_n}(z),\ \in_{ij}=\sum_{n=1}^{N}\in_{ij}^{n}\pi_{h_{n-1},h_n}(z),\ \rho=\sum_{n=1}^{N}\rho^{n}\pi_{h_{n-1},h_n}(z),\\
&q_{ij}=\sum_{n=1}^{N}q_{ij}^{n}\pi_{h_{n-1},h_n}(z),\ g_{ij}=\sum_{n=1}^{N}g_{ij}^{n}\pi_{h_{n-1},h_n}(z),\ \mu_{ij}=\sum_{n=1}^{N}\mu_{ij}^{n}\pi_{h_{n-1},h_n}(z).
\end{aligned} \tag{5b}$$