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Stacking sequence optimization for maximum strengths of laminated composite plates using genetic algorithm and isogeometric analysis

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ABSTRACT

In this paper, an optimal procedure based on genetic algorithm and NURBS-based finite element isogeometric analysis (IGA) is proposed for seeking maximum load-carrying capacity of imperfect laminated composite plates with respect to a specified displacement. Nonlinear behavior of plates including bending, buckling and postbuckling are investigated. Fiber orientations are the design variables. Governing equations are derived in the framework of first-order shear deformation theory (FSDT) associated with von-Karman strain theory for large deformation. Isogeometric analysis is known as a robust numerical method in dealing with laminated structures due to the smoothness, high continuity and the reduction of total degrees-of-freedom (DOFs). Modified Riks method is employed for nonlinear analysis. The arc-length scheme simultaneously adjusts displacement and load so that accelerates the analysis procedure. Computational time is significantly reduced based on the cooperation of IGA and Riks algorithm. Numerical examples including 4-ply and quasi symmetric laminates are investigated to show the efficiency and flexibility of the proposed approach.

1. Introduction

Genetic algorithms (GAs) are well-known as probabilistic searching methods based on Darwin's theory about evolution in which the best-fitted members of population transmit their attractive features to next generation and new features are created by genetic mutation. GAs are non-gradient-based methods so that suitable for discrete optimization problems. In addition, GAs are insensitive to large and complex design variables. GAs have been becoming popular since developed by Rechenberg [1] and Holland [2]. However, the standard GAs usually requires a high computational cost. Therefore, several modifications for particular problems were carried out in order to reduce computational time [3–10]. In optimizing laminate stacking sequences, number of layers, fiber orientation and thickness of each layer are usually chosen as design variables with various objective functions such as strength [11], buckling load [12], stiffness [13]. A recent review study of Ghiasi et al. [14] provides a systematic summary of optimization methods including GA developments as well as applications.

Isogeometric finite element analysis (IGA) was introduced in 2005 by Hughes et al. [15] and received numerous attentions due to its significant advantages. The main feature is that NURBS (Non-Uniform Rational B-Spline) basis functions are used as interpolating functions to construct geometry and approximate displacement field simultaneously. Beside the advantages of smoothness and high continuity of geometry, total degrees-of-freedom (DOFs) is significantly reduced in comparison with regular finite element methods. In a recent book, Hughes and collaborators [16] summarized their works on IGA, presented a systematic procedure to apply IGA in structural analysis, fluid mechanics and electromagnetic. Three versatile global refinement methods such as the h-refinement of knot insertion, the p-refinement of order elevation and the k-refinement for higher order and higher continuity were introduced [15,17,16]. Local refinement methods were developed with T-Splines [18–21] and PHT-Splines [22]. Efficient quadrature rules for IGA framework were considered [23,24]. Locking and unlocking of NURBS finite elements were also investigated [25]. Studies for free vibration, buckling, geometrically nonlinear and postbuckling behavior of composite laminates based on the combination of IGA and FSDT, HSDT, layerwise theory were carried out [26–34].

Numerous studies combined GAs with a finite element analysis (FEA) procedure to investigate stacking sequence optimization of laminated composite plates for various purposes of maximizing strength, stiffness, minimizing weight, deflection or multiple objectives. However, almost studies employed linear FEA to derive

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the objective functions due to the large number of iterations in GA analysis process. In case of considering nonlinear analysis, the FEA procedure should be improved to reduce computational cost as much as possible. Isogeometric analysis method provides an effective solution based on the significant reduction of DOFs in NURBS meshes. Therefore, an optimal procedure using GAs combined with IGA is proposed to optimize laminate stacking sequences for maximum nonlinear bending strength, buckling load and postbuckling strength. Additionally, modified Riks scheme instead of the conventional Newton–Raphson method is employed for nonlinear analysis. The arc-length scheme simultaneously adjusts displacement and load so that speeds up the analysis procedure.

The next Section gives a brief of first-order shear deformable laminated plate theory using in this work. Then NURBS-based isogeometric analysis for 2D problems is represented in Section 3. Formulations for genetic algorithm optimal process is provided in Section 4. 4-ply and quasi symmetric laminates are investigated in numerical example section to demonstrate the efficiency and flexibility of the proposed approach. Conclusions are remarked in the final section.

2. Theoretical formulation

Shear deformation theories for composite plates could be found in the excellent text books of Reddy [35–37]. A brief of formulations which are used in this work would be carried out in this section. Governing equations for nonlinear static problems are constructed by using FSDT associated with von-Karman strain tensor. Imperfection applied on perfectly flat plates in postbuckling analysis was investigated by Le-Manh and Lee [30]. Imperfections are set to zero in nonlinear bending analysis.

2.1. Strain–displacement relations

Fig. 1 illustrates a geometry of laminated composite plates in which the origin of material coordinate system is considered as the mid-plane. In displacement field, a point in domain Ω is described by a point in mid-plane Ω_0 and the rotations as follows,

$$U_1(x_1,x_2,x_3) = u_1(x_1,x_2) + x_3\phi_1(x_1,x_2) \tag{1a}$$
$$U_2(x_1,x_2,x_3) = u_2(x_1,x_2) + x_3\phi_2(x_1,x_2) \tag{1b}$$
$$U_3(x_1,x_2,x_3) = u_3(x_1,x_2) \tag{1c}$$

where U_i and u_i are the displacement of a point in domain and the displacement of a mid-plane point along the x_i direction respectively, ϕ_1 and ϕ_2 are the rotations of transverse displacement of the mid-plane about the x_2 and x_1 axis respectively.

Imperfection defined as an initial deviation from being perfectly flat is also a function of mid-plane coordinates, $\mu(x_1,x_2)$. Deformed geometry of a plate including imperfections in FSDT is shown in Fig. 2.

Nonlinear strain components associated with displacement field for large deformation of large deflection, small inplane strains and small rotations are computed based on von-Karman theory,

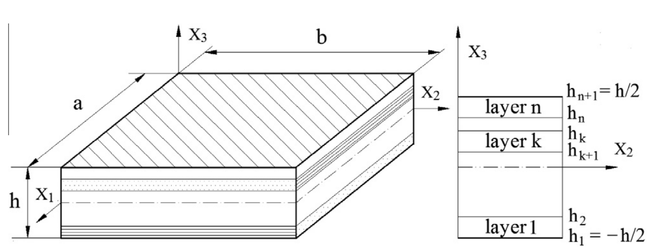

Fig. 1. Geometry of laminated composite plates.

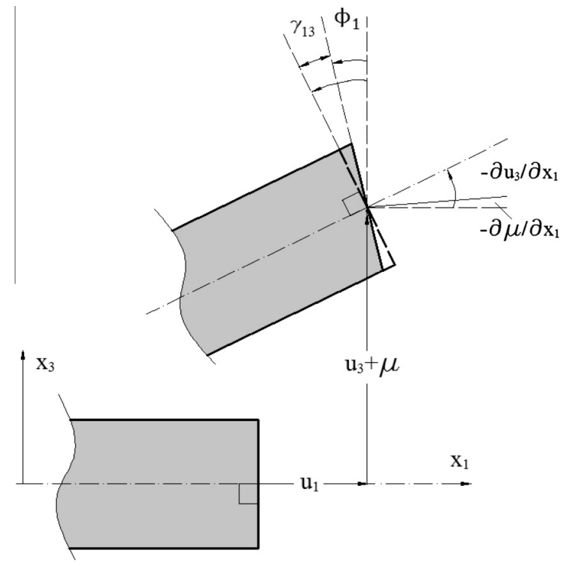

Fig. 2. Deformed geometry of an imperfect plate in FSDT.

$$\epsilon_{11} = u_{1,1} + \frac{1}{2}u_{3,1}^2 + x_3\phi_{1,1} \tag{2a}$$
$$\epsilon_{22} = u_{2,2} + \frac{1}{2}u_{3,2}^2 + x_3\phi_{2,2} \tag{2b}$$
$$\gamma_{23} = u_{3,2} + \phi_2 \tag{2c}$$
$$\gamma_{13} = u_{3,1} + \phi_1 \tag{2d}$$
$$\gamma_{12} = u_{1,2} + u_{2,1} + u_{3,1}u_{3,2} + x_3(\phi_{1,2} + \phi_{2,1}) \tag{2e}$$

The strain components are modified for imperfections and rearranged in terms of extensional strains, bending shear strains, and mid-plane curvatures respectively as follows,

$$\epsilon_{\alpha\beta}^0 = \frac{1}{2}\left(u_{\alpha,\beta} + u_{\beta,\alpha} + u_{3,\alpha}u_{3,\beta} + u_{3,\alpha}\mu_{,\beta} + u_{3,\beta}\mu_{,\alpha}\right) \tag{3}$$
$$\gamma_{\alpha3}^0 = u_{3,\alpha} + \mu_{,\alpha} + \phi_\alpha \tag{4}$$
$$\kappa_{\alpha\beta} = \frac{1}{2}(\phi_{\alpha,\beta} + \phi_{\beta,\alpha}) \tag{5}$$

where Greek subscripts α and β represent the integer values from 1 to 2. The curvatures caused by imperfections could be also neglected due to their small magnitude.

2.2. Governing equations

The inplane force resultants $N_{\alpha\beta}$, moment resultants $M_{\alpha\beta}$ and transverse shear force resultants Q_α are defined as follows,

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\epsilon}^0 \\ \boldsymbol{\kappa} \end{Bmatrix} \tag{6}$$
$$\mathbf{Q} = \mathbf{A}^s\boldsymbol{\gamma}^0 \tag{7}$$

where, $\mathbf{A}, \mathbf{B}, \mathbf{D}$ and $\mathbf{A}^s$ represent extensional stiffness matrix, extensional-bending coupling stiffness matrix, bending stiffness matrix and transverse shearing stiffness matrix, respectively.

The governing equations could be derived based on principle of virtual work and finally formed in terms of resultants and applied forces as follows,

$$\begin{aligned} &N_{11,1} + N_{12,2} = 0 \\ &N_{12,1} + N_{22,2} = 0 \\ &Q_{1,1} + Q_{2,2} + \aleph + q = 0 \\ &M_{11,1} + M_{12,2} - Q_1 = 0 \\ &M_{12,1} + M_{22,2} - Q_2 = 0 \end{aligned} \tag{8}$$