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Numerical investigations of effective properties of fiber reinforced composites with parallelogram arrangements and imperfect interface

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ABSTRACT

In this paper unidirectional fiber composites with imperfect interface conditions between the reinforcement and the filler are considered. The microstructure is periodic and the phases have isotropic properties. The periodicity of the microstructure is characterized by a parallelogram. Using the concept of a representative volume element (RVE) a finite element model is developed, in which the distribution of fibers and imperfect contact conditions between interfaces of phases can vary. Applying appropriate periodic boundary conditions to the chosen RVE effective material properties are derived, where those are related to a predefined coordinate system. The homogenization technique is validated by comparing results to literature as far as possible.

1. Introduction

The research on techniques to determine or approximate effective material properties of composites is a large field. For a two phase composite with randomly distributed inclusions, whose heterogeneous material behavior differs from a homogeneous one, several analytical procedures stated in literature can be used to approximate the effective coefficients. Famous methods are the Mori–Tanaka-method [18], the generalized self-consistent scheme [5,6,13] and the bounds of Hashin and Shtrikman [8,9]. Considering additional assumptions with respect to the material and the geometry of the inclusion phase closed-form expressions for the coefficients are derivable.

With the initiation of the computer technology the concept of the representative volume element (RVE) in combination with a finite element analysis, given for instance by the software package ANSYS, gets more and more importance [4,15,21]. By this combination it is possible to observe periodic and complex structures as well.

An often used assumption is the perfect bonding of the material phases, which means continuity in displacements and stresses at the interface of both phases. Due to production processes it could be more realistic to consider a three phase composite, where the third phase deals as coating or adhesion. For instances in Hashin [13], Andrianov et al. [2], Kari et al. [16] and Yan et al. [24] effective properties for three phase composites are presented. Assuming a constant and small thickness of such a coating, the third phase, also called interphase, can be treated as an interface, which fulfills appropriate conditions. Such an interface is also called imperfect interface, since the conditions on the interface consist of jumps in physical quantities as for instance stresses and displacements.

Often used conditions at the interface are that the traction components of the constituents of the composite are defined by differences or jumps of displacements multiplied with proportionality factors. This factor is in general given by a constant, which has N/mm^3 as unit of measurement. This type of contact has been studied for instance by Benveniste [3], Hashin [11–13], López-Realpozo et al. [17], Andrianov et al. [1] and Otero et al. [19]. In some of these articles effective material properties are presented.

In this paper a model based on a numerical homogenization technique is developed, in which unidirectional fiber composites are considered, where the fibers are arranged in such a pattern, that a spatial domain with a parallelogram shaped cross section periodically repeated represents the composite structure. Such composite structures for the case of perfect bonding between constituents are studied by Golovchan and Nikityuk [7] and Rodríguez-Ramos et al. [20], where only effective out-of-plane coefficients are presented. By the parametric algorithm rhombic, hexagonal, square or rectangular fiber arrangements are considerable. So a wider class of composite structures can be observed. In addition the model also takes into account imperfect contact behavior between the fiber and matrix phase. So the elastic interactions between the phases can be influenced through contact parameters. This paper can be treated as an extension of a previous work of Würkner et al. [23], where fiber reinforced composites

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with a microstructure characterized by a rhombic cross section and an imperfect contact are observed. For verification of the present model effective composite properties presented within this paper are compared to others reported in the given literature.

2. Numerical procedure and imperfect modeling

In this section fundamentals of a homogenization procedure based on the elastic theory and the development of an appropriate representative volume element (RVE) are presented in order to create a finite element model, which consists of two different material phases (matrix, fiber) and an imperfect interface.

2.1. Homogenization method

For the research within this paper a two phase model is considered, which consists of a fiber and a matrix phase, where all fibers have the same radius and they are unidirectional orientated. The fibers have a circular cross section. In addition with respect to the transversal plane they are arranged in such a pattern, that a parallelogram characterizes the periodic microstructure. In Fig. 1 the chosen RVE and the considered coordinate system y_1, y_2 (y_3 is directed out of the plane) can be seen, where the direction of the coordinates are also used for the representation of the stiffness tensor coefficients in the result section. The cross section geometry of the RVE is defined by the angle α and the length of the edges l_1 and l_2. Through varying these parameters in a computer routine it is possible to construct RVEs characterizing composites with special fiber arrangements. These cases are presented in Fig. 2. On the left side a rectangular shaped RVE is shown, where $\alpha = 90°$ and $l_1 \neq l_2$. In case of equality of the length parameters square fiber arrangements are achieved. In the center of the figure the case of a rhombic shaped RVE can be seen, where $0° < \alpha < 90°$ and $l_1 = l_2$. This configuration includes a hexagonal fiber distribution with an angle of 60°. On the right side of the figure an RVE with an arbitrary parallelogram shaped cross section with $0° < \alpha < 90°$ and $l_1 \neq l_2$ is presented.

The constitutive law in each phase of the composite is given by the Hooke's law, which is written in Einstein summation convention as follows

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}, \quad i,j,k,l = 1,2,3, \tag{1}$$

where σ_{ij}, ε_{kl} and C_{ijkl} are the coefficients of the stress tensor, the linear strain tensor and the stiffness tensor, respectively. The subscripts are related to a previously chosen Cartesian coordinate system, which is parallel to the system shown in Fig. 1. It is stated, that the coefficients of Eq. (1) fulfill the symmetry conditions of the linear elasticity [22] and in addition in case of the stiffness coefficients the symmetry properties due to isotropic or transversal isotropic material considerations.

By homogenization techniques in order to evaluate effective properties it is sufficient to consider a periodic microstructure denoted as V_{rve}, which forms the RVE shown in Fig. 1. Due to linear elasticity in each phase the following equations of equilibrium

$$\frac{\partial}{\partial y_j}\sigma_{ij}(\mathbf{y}) = 0, \tag{2}$$

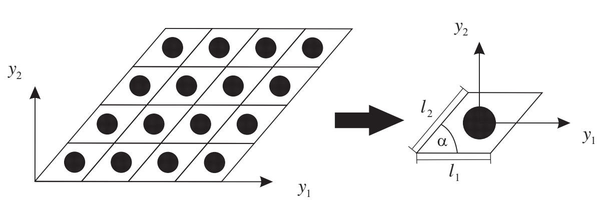

Fig. 1. Composite structure and considered RVE with geometric parameters.

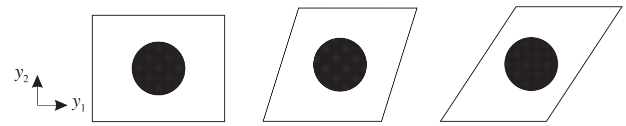

Fig. 2. Possible cases for cross sections of the RVE modeled by the present algorithm, left: rectangle, middle: rhomb, right: parallelogram.

hold. At the interface Γ, which connects the fiber and the matrix phase, the imperfect contact conditions are given by

$$\begin{aligned}
\sigma^m_{rr} &= \sigma^f_{rr} = K_r\|u_r\|,\\
\sigma^m_{r\theta} &= \sigma^f_{r\theta} = K_\theta\|u_\theta\|,\\
\sigma^m_{rz} &= \sigma^f_{rz} = K_z\|u_z\|,
\end{aligned} \tag{3}$$

where the values σ^i_{rr}, $\sigma^i_{r\theta}$ and σ^i_{rz} are the surface traction components with respect to a cylindrical coordinate system, which are related to either the fiber ($i = f$) or the matrix phase ($i = m$). The quantities K_i, $i = r, \theta, z$ are spring type parameters, which have the dimension of stresses divided by length. The double bar $\|\cdot\|$ defines differences of displacements between matrix and fiber phase

$$\|u_i\| = u^m_i - u^f_i, \quad i = r,\ \theta,\ z. \tag{4}$$

The parameters K_i, $i = r, \theta, z$ represent the elastic behavior of the interface transferring loads between phases. Due to previous investigations [13] these quantities can be identified from a three phase problem, where the third phase coating the fiber is very thin. In the case of an isotropic interphase, they have the form

$$\begin{aligned}
K_r &= \frac{E^i(1-\nu^i)}{t(1-2\nu^i)(1+\nu^i)},\\
K_\theta &= \frac{G^i}{t},\\
K_z &= \frac{G^i}{t},
\end{aligned} \tag{5}$$

where E^i, ν^i and t are the Young's modulus, Poisson's ratio and the radial directional thickness of the interphase, respectively.

For a complete description of a differential problem in order to determine effective material properties it is necessary to formulate appropriate boundary conditions. Since periodic structures are investigated so called periodic boundary conditions are applied to the considered RVE. These conditions are derived from the assumption, that the displacements on the boundary ∂V_{rve} are a superposition of a linear and a periodic part u^{per}_i

$$u_i(\mathbf{y}) = \varepsilon^0_{ij} y_j + u^{per}_i(\mathbf{y}), \tag{6}$$

where ε^0_{ij} are given quantities [21]. The quantities $\mathbf{y}$ and y_j are the vector of location and the jth component of it, respectively. As in the case of perfect contact condition it is assumed, that the averages of stresses and strains representing the stresses and strains of the composite on macro level are given by

$$\langle\sigma_{ij}\rangle = \frac{1}{|V_{rve}|}\int_{\partial V_{rve}} \sigma_{ir} n_r y_j\, dS, \tag{7}$$

$$\langle\varepsilon_{kl}\rangle = \frac{1}{|V_{rve}|}\int_{\partial V_{rve}} u_k n_l + u_l n_k\, dS, \tag{8}$$

where n_i is the ith component of the outer normal vector of the RVE [3]. The effective coefficients of the composite C^{eff}_{ijkl} can be derived from the relation

$$\langle\sigma_{ij}\rangle = C^{eff}_{ijkl}\langle\varepsilon_{kl}\rangle, \tag{9}$$

which represents the constitutive law of the homogenized composite material. From the Eqs. (7) and (8) it can be shown, that the relations