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Homogenization of periodic hexa- and tetrachiral cellular solids

Andrea Bacigalupo [a], Luigi Gambarotta [b,*]

[a] Department of Civil, Environmental and Mechanical Engineering, University of Trento, Via Mesiano, 77, 38123 Trento, Italy
[b] Department of Civil, Chemical and Environmental Engineering, University of Genoa, Via Montallegro, 1, 16145 Genoa, Italy

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ABSTRACT

The homogenization of periodic hexachiral and tetrachiral honeycombs is dealt with two different techniques. The first is based on a micropolar homogenization. The second approach, developed to analyse two-dimensional periodic cells consisting of deformable portions such as the ring, the ligaments and possibly a filling material, is based on a second gradient homogenization developed by the authors. The obtained elastic moduli depend on the parameter of chirality, namely the angle of inclination of the ligaments with respect to the grid of lines connecting the centers of the rings. For hexachiral cells the auxetic property of the lattice together with the elastic coupling modulus between the normal and the asymmetric strains is obtained; a property that has been confirmed here for the tetrachiral lattice. Unlike the hexagonal lattice, the classical constitutive equations of the tetragonal lattice turns out to be characterized by the coupling between the normal and shear strains through an elastic modulus that is an odd function of the parameter of chirality. Moreover, this lattice is found to exhibit a remarkable variability of the Young's modulus and of the Poisson's ratio with the direction of the applied uniaxial stress. Finally, a simulation of experimental results is carried out.

1. Introduction

Auxetic materials, having zero or negative Poisson's ratio, are characterized by non-conventional mechanical response with respect to many common materials: they become thicker widthwise when stretched lengthwise and thinner when compressed [1]. Although some natural materials may be classified as auxetic, this quality is mostly obtained in man-made materials [2]. This unusual mechanical behavior may result in an increased stiffness and indentation resistance of the auxetic material and a higher toughness due to an increase of the energy absorption under static and dynamic loading, thus making these smart materials of special technological interest (see [3]). The auxetic effect occurs in cellular materials, such as foams (see [4]), honeycomb structures and networks [5] and origami structures [6], as the result of the unfolding of re-entrant cells as they are stretched. The design of auxetic materials is mostly addressed to periodic cellular composites [7] through the analysis and optimization of periodic manufacturable cells [8,9]. In addition to the periodic microstructures based on re-entrant mechanisms, auxetic materials based on mechanisms of rotating rigid and semi-rigid units [10] and on rolling-up mechanisms [11] have been developed. This latter mechanism occurs in two-dimensional honeycomb structures composed of circular rings periodically located in the material plane and joined by straight ligaments characterized by chiral (see Fig. 1) or anti-chiral topologies.

Alderson et al. [12,13], carried out experiments on samples having both chiral and anti-chiral periodic cells subjected to uniaxial compression, together with numerical simulations of the experimental results obtained by a standard FE homogenization of the periodic cell. While a rather good agreement in the overall elastic moduli was found for the hexachiral cell (Fig. 1(a)), qualitative differences were obtained between the experimental and numerical results for the tetrachiral cell. Further theoretical and experimental analyses have been carried out by Lorato et al. [14] and Cicala et al. [15], to investigate the transverse elastic properties of chiral honeycombs, and by Chen et al. [16], to derive the in-plane elastic moduli of anti-tetrachiral lattices. Moreover, Ma et al. [17], have performed static and dynamic testing on anti-tetrachiral honeycombs, with rings filled with metal rubber particles in order to obtain prescribed dynamic performances.

With reference to the chiral topologies, the study of the mechanical behavior of hexachiral structures started from the seminal paper by Prall and Lakes [11], and was later developed to include the analysis of damage processes (see [18]) and of free wave propagation [19]. Afterwards, Tee et al. [20], applied a FE analysis based on the Floquet–Bloch approach to obtain the

* Corresponding author. Tel.: +39 0103532556.
E-mail address: luigi.gambarotta@unige.it (L. Gambarotta).

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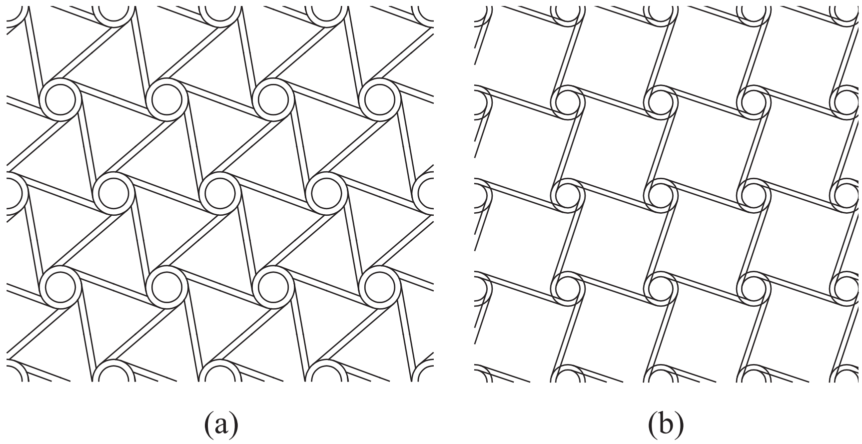

Fig. 1. (a) Hexachiral lattice; (b) tetrachiral lattice.

phononic properties of the tetrachiral periodic cell. Gatt et al. [21], proposed a homogenization model for anti-tetrachiral systems based on a beam-lattice model and verified its reliability through a FE modeling of the periodic cell. An analysis of the overall elastic properties of chiral and anti-chiral cellular solids was carried out by Dirrenberger et al. [22,23], that is based on the classical homogenization approach through a finite element discretization of the periodic cell. This approach has been extended by Dirrenberger et al. [24], to the analysis of the elasto-plastic response of hexachiral ductile materials.

Spadoni and Ruzzene [25], developed a micropolar homogenization of the hexachiral beam-lattice model (see Fig. 2(b)) based on the approach of Kumar and McDowell [26], that is equipped with an internal length directly associated with the characteristic size of the microstructure. The overall elastic moduli of the equivalent micropolar continuum were found to depend on the chirality parameter β measured by the angle of inclination of the ligaments with respect to the grid of lines connecting the centers of the rings. Moreover, the classical elastic moduli of the equivalent transversely isotropic continuum, i.e., the overall Young's modulus and Poisson's ratio, were derived so improving the estimation of the Poisson's ratio obtained from Prall and Lakes [11]. A further improvement of the micropolar homogenization of hexachiral beam-lattice was obtained by Liu et al. [27], which have shown that the chiral geometry determines a coupling between the bulk deformation and the pure rotation. This effect is described by an elastic modulus that is an odd function of the chirality angle β, namely it reverses its sign when the material pattern is flipped over. Despite this improvement of the micropolar model, the resulting overall elastic moduli of the classical continuum are unchanged from those obtained by Spadoni and Ruzzene [25].

The results by Liu et al. [27], Spadoni and Ruzzene [25], concerning the hexachiral beam-lattice model (Fig. 2(b)) provide a richer description of the dependence of the elastic moduli on the chirality and deserve to be extended to the tetrachiral geometry (Fig. 2(b)). On the other hand, the beam-lattice model can be regarded as appropriate for very slender ligaments, a circumstance that does not seem to occur in some samples used in experiments where the effective length of the ligament is not easy to identify. Furthermore, this model does not include the presence of the filling material between the ligaments and inside of the rings. For these reasons it seems necessary to define an equivalent continuum at the macroscale, preferably a non-local continuum equipped with internal lengths, which is based on a FE description of the periodic cell. Although a smart technique for micropolar homogenization of two-dimensional cells [28] is available, there are considerations that limit its use (see [29]) and suggest computational homogenization techniques based on second gradient continuum models (see [30–34]). To support this choice, some studies aimed to define the elastic properties of chiral materials according to the strain gradient elasticity [35–38] may be regarded as a reference for the validation of numerical simulations.

In this paper the overall elastic moduli for both hexachiral and tetrachiral periodic cells (Figs. 2 and 3) are obtained with reference to both the micropolar and second displacement gradient continuum models. At first, the cellular materials are modeled as beam lattices having rigid circular rings and elastic beams with rigid ends to represent the ligaments and a micropolar equivalent

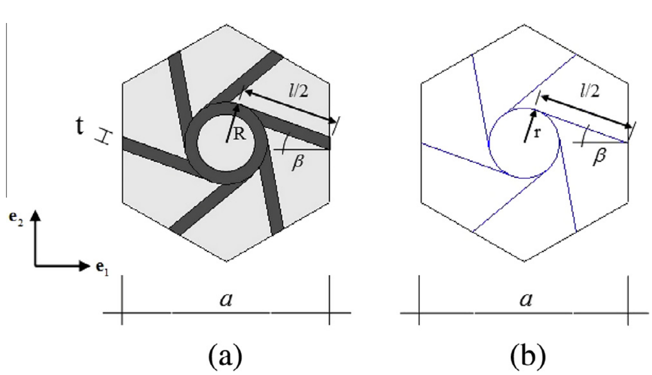

Fig. 2. Hexachiral periodic cells: (a) two-dimensional; (b) beam-lattice.

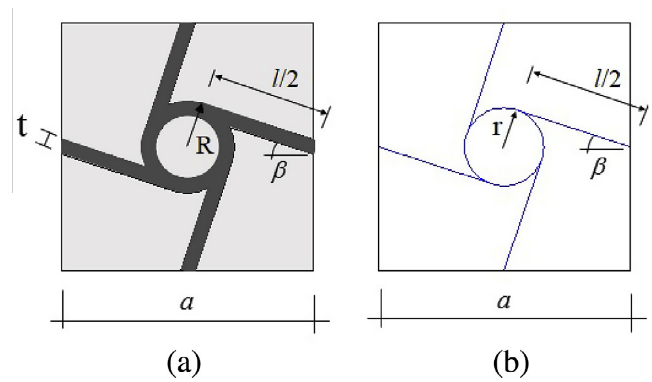

Fig. 3. Tetrachiral periodic cells: (a) two-dimensional; (b) beam-lattice.