Composite Structures 116 (2014) 477–488

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Torsional postbuckling of nanotube-reinforced composite cylindrical shells in thermal environments

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ARTICLE INFO

Article history:
Available online 10 June 2014

Keywords:
Nanocomposites
Functionally graded materials
Cylindrical shell
Postbuckling
Torsion

ABSTRACT

A postbuckling analysis is presented for a functionally graded composite cylindrical shell reinforced by single-walled carbon nanotubes (SWCNTs) subjected to torsion in thermal environments. The multi-scale model for functionally graded carbon nanotube-reinforced composite (FG-CNTRC) shells under torsion is proposed. A singular perturbation technique along with a two-step perturbation approach is employed to determine the buckling load and postbuckling equilibrium path. The numerical illustrations concern the torsional buckling and postbuckling behavior of perfect and imperfect, FG-CNTRC cylindrical shells under different sets of thermal environmental conditions. The results for uniformly distributed CNTRC shell, which is a special case in the present study, are compared with those of the FG-CNTRC shell. The results show that the linear functionally graded reinforcements can increase the buckling torque as well as postbuckling strength of the shell under torsion when the reinforcement has a symmetrical distribution. The results reveal that the carbon nanotube volume fraction has a significant effect on the buckling load and postbuckling behavior of CNTRC shells under torsion.

1. Introduction

The postbuckling behavior of functionally graded carbon nanotube-reinforced composite (CNTRC) cylindrical shells subjected to either axial compression or external pressure in thermal environments was the subject of a recent investigation [1,2]. A functionally graded CNT reinforced aluminum matrix composite was recently fabricated by a powder metallurgy route to support the concept of functionally graded materials in the nanocomposites [3]. Consequently, investigations on the bending, buckling and vibration behaviors of functionally graded CNTRC beams [4–10], plates [11–19], shells [20–24] and panels [25–29] are identified as an interesting field of study in recent years.

Torsional buckling analysis is a difficult task for cylindrical shells. This is due to the fact that the solution becomes more complicated in the case of cylindrical shells under torsion. The notable contributions pertaining to the torsional postbuckling analysis of isotropic and composite cylindrical shells are available in Loo [30], Nash [31], Yamaki and Matsuda [32], Chehil and Cheng [33]. Recently, Shen [34] presented the postbuckling behavior of FGM cylindrical shells subjected to torsion in thermal environments. In his study, the material properties were considered to be temperature-dependent and the effect of temperature rise and/or heat conduction on the postbuckling behavior was reported. It is concluded that the torsional postbuckling equilibrium path of moderately long FGM cylindrical shells is weakly unstable and the shell structure is virtually imperfection-insensitive.

For an isotropic cylindrical shell under torsion, the classical solutions proposed by Loo [30] and Nash [31] are formed as

$$\overline{W} = W_1 \sin(\pi X/L)\sin(nY/R + knX/R) \tag{1}$$

$$\overline{W} = W_1[1 - \cos(2\pi X/L)]\sin(nY/R + knX/R) \tag{2}$$

where the parameter k is determined by minimizing the strain energy. Loo's solution was also adopted by Chehil and Cheng [33] for a composite laminated cylindrical shell and by Huang and Han [35] for an FGM cylindrical shell. It is worth noting that both Eqs. (1) and (2) can not satisfy boundary conditions such as simply supported or clamped at the end of the cylindrical shell and may then be as approximate solutions.

It has been shown [1], the governing equations for an CNTRC cylindrical shell are identical in form to those of unsymmetric cross-ply laminated cylindrical shells. Tabiei and Simitses [36] attempted to give more accurate solutions for laminated cylindrical shells under torsion. They suggested solutions formed as

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http://dx.doi.org/10.1016/j.compstruct.2014.05.039

$$\overline{W}=\sum_{m=1}\sum_{n=0}\left[W_{mn}\sin\frac{nY}{R}+W'_{mn}\cos\frac{nY}{R}\right]\left[\cos\frac{m\pi X}{L}-\cos\frac{(m-2)\pi X}{L}\right] \quad (3)$$

Since sufficient numbers of unknown parameters are retained, the solutions of Eq. (3) could satisfy both compatibility and boundary conditions, but they do not satisfy equilibrium equation, and therefore, the Galerkin method had to be performed.

It has been reported [37] that, there exists a shear stress along with an associate compressive stress when the anisotropic shell is subjected to torsion. This will help us to have a better understanding of the mechanism for CNTRC cylindrical shells under torsion.

In the present study, the nanocomposite shells are assumed to be functionally graded in the thickness direction using single-walled carbon nanotubes (SWCNTs) serving as reinforcements and the shells' effective material properties are estimated through a micromechanical model, in which the CNT efficiency parameter is estimated by matching the elastic modulus of CNTRCs observed from the molecular dynamics (MD) simulation results with the numerical results obtained from the extended rule of mixture. The governing equations are based on a higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity and include thermal effects. A singular perturbation technique along with a two-step perturbation approach is employed to determine the buckling torque and postbuckling equilibrium path. The nonlinear prebuckling deformations and initial geometric imperfections of the shell are both taken into account. The numerical illustrations show the full nonlinear postbuckling response of CNTRC cylindrical shells subjected to torsion in environmental conditions.

2. Multi-scale model for functionally graded CNTRC shells under torsion

Consider an CNTRC cylindrical shell with mean radius R, length L and thickness h. The shell is referred to a coordinate system (X, Y, Z) in which X and Y are in the axial and circumferential directions of the shell and Z is in the direction of the inward normal to the middle surface. The corresponding displacements are designated by $\overline{U}$, $\overline{V}$ and $\overline{W}$. $\overline{\Psi}_x$ and $\overline{\Psi}_y$ are the rotations of the normals to the middle surface with respect to the Y and X axes, respectively. The origin of the coordinate system is located at the end of the shell in the middle plane. The shell is assumed to be geometrically imperfect, and is subjected to a torque uniformly applied along the edges. Denoting the initial geometric imperfection by $\overline{W}^*(X,Y)$, let $\overline{F}(X,Y)$ be the stress function for the stress resultants defined by $\overline{N}_x=\overline{F}_{,YY}$, $\overline{N}_y=\overline{F}_{,XX}$ and $\overline{N}_{xy}=-\overline{F}_{,XY}$, where a comma denotes partial differentiation with respect to the corresponding coordinates.

Reddy and Liu [38] developed a simple higher order shear deformation shell theory. This theory was a modification of the Sanders shell theory and accounts for parabolic distribution of the transverse shear strains through the thickness of the shell and tangential stress-free boundary conditions on the boundary surfaces of the shell. As has been shown [34] this theory can accurately predict the torsional buckling of single layer FGM shells. The advantages of this theory over the first order shear deformation theory are that the number of independent unknowns ($\overline{U},\overline{V},\overline{W},\overline{\Psi}_x$ and $\overline{\Psi}_y$) is the same as in the first order shear deformation theory, but no shear correction factors are required. Based on Reddy's higher order shear deformation theory with a von Kármán-type of kinematic nonlinearity and including thermal effects, the governing differential equations for an FG-CNTRC cylindrical shell can be derived in terms of a stress function $\overline{F}$, two rotations $\overline{\Psi}_x$ and $\overline{\Psi}_y$, and a transverse displacement $\overline{W}$, along with the initial geometric imperfection $\overline{W}^*$. They are

$$\tilde{L}_{11}(\overline{W})-\tilde{L}_{12}(\overline{\Psi}_x)-\tilde{L}_{13}(\overline{\Psi}_y)+\tilde{L}_{14}(\overline{F})-\tilde{L}_{15}(\overline{N}^T)-\tilde{L}_{16}(\overline{M}^T)-\frac{1}{R}\overline{F}_{,XX}=\tilde{L}(\overline{W}+\overline{W}^*,\overline{F}) \quad (4)$$

$$\tilde{L}_{21}(\overline{F})+\tilde{L}_{22}(\overline{\Psi}_x)+\tilde{L}_{23}(\overline{\Psi}_y)-\tilde{L}_{24}(\overline{W})-\tilde{L}_{25}(\overline{N}^T)+\frac{1}{R}\overline{W}_{,XX}=-\frac{1}{2}\tilde{L}(\overline{W}+2\overline{W}^*,\overline{W}) \quad (5)$$

$$\tilde{L}_{31}(\overline{W})+\tilde{L}_{32}(\overline{\Psi}_x)-\tilde{L}_{33}(\overline{\Psi}_y)+\tilde{L}_{34}(\overline{F})-\tilde{L}_{35}(\overline{N}^T)-\tilde{L}_{36}(\overline{S}^T)=0 \quad (6)$$

$$\tilde{L}_{41}(\overline{W})-\tilde{L}_{42}(\overline{\Psi}_x)+\tilde{L}_{43}(\overline{\Psi}_y)+\tilde{L}_{44}(\overline{F})-\tilde{L}_{45}(\overline{N}^T)-\tilde{L}_{46}(\overline{S}^T)=0 \quad (7)$$

in which

$$\tilde{L}(\,)=\frac{\partial^2}{\partial X^2}\frac{\partial^2}{\partial Y^2}-2\frac{\partial^2}{\partial X\partial Y}\frac{\partial^2}{\partial X\partial Y}+\frac{\partial^2}{\partial Y^2}\frac{\partial^2}{\partial X^2} \quad (8)$$

and the other linear operators $\tilde{L}_{ij}(\,)$ are defined as in Shen [1]. Note that the geometric nonlinearity in the von Kármán sense is given in terms of $\tilde{L}(\,)$ in Eqs. (4) and (5).

The temperature field is assumed to be a uniform distribution over the shell surface and through the shell thickness. In the above equations, $\overline{N}^T$, $\overline{M}^T$, $\overline{S}^T$, and $\overline{P}^T$ are the forces, moments and higher order moments caused by elevated temperature, and are defined by

$$\begin{bmatrix}\overline{N}_x^T & \overline{M}_x^T & \overline{P}_x^T\\ \overline{N}_y^T & \overline{M}_y^T & \overline{P}_y^T\\ \overline{N}_{xy}^T & \overline{M}_{xy}^T & \overline{P}_{xy}^T\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}A_x\\ A_y\\ A_{xy}\end{bmatrix}(1,Z,Z^3)\Delta T dZ \quad (9a)$$

$$\begin{bmatrix}\overline{S}_x^T\\ \overline{S}_y^T\\ \overline{S}_{xy}^T\end{bmatrix}=\begin{bmatrix}\overline{M}_x^T\\ \overline{M}_y^T\\ \overline{M}_{xy}^T\end{bmatrix}-\frac{4}{3h^2}\begin{bmatrix}\overline{P}_x^T\\ \overline{P}_y^T\\ \overline{P}_{xy}^T\end{bmatrix} \quad (9b)$$

where $\Delta T = T - T_0$ is the temperature rise from some reference temperature T_0 at which there are no thermal strains, and

$$\begin{bmatrix}A_x\\ A_y\\ A_{xy}\end{bmatrix}=-\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\alpha_{11}\\ \alpha_{22}\end{bmatrix} \quad (10)$$

where α_{11} and α_{22} are the thermal expansion coefficients measured in the longitudinal and transverse directions, and $\overline{Q}_{ij}$ are the transformed elastic constants with details being given in [38]. Note that for an FG-CNTRC layer, $\overline{Q}_{ij}=Q_{ij}$ in which

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}},\quad Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}},\quad Q_{12}=\frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}},$$
$$Q_{16}=Q_{26}=0,\quad Q_{44}=G_{23},\quad Q_{55}=G_{13},\quad Q_{66}=G_{12} \quad (11)$$

where E_{11}, E_{22}, G_{12}, ν_{12} and ν_{21} are the effective Young's and shear moduli and Poisson's ratio of the FG-CNTRC layer, respectively. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model, as expressed by [11]

$$E_{11}=\eta_1 V_{CN}E_{11}^{CN}+V_mE^m \quad (12a)$$

$$\frac{\eta_2}{E_{22}}=\frac{V_{CN}}{E_{22}^{CN}}+\frac{V_m}{E^m} \quad (12b)$$

$$\frac{\eta_3}{G_{12}}=\frac{V_{CN}}{G_{12}^{CN}}+\frac{V_m}{G^m} \quad (12c)$$

where E_{11}^{CN}, E_{22}^{CN} and G_{12}^{CN} are the Young's and shear moduli of the CNTs, E^m and G^m are the corresponding properties for the matrix, and the $\eta_j (j = 1,2,3)$ are the CNT efficiency parameters, respectively. In addition, V_{CN} and V_m are the volume fractions of the CNT and the matrix, which satisfy the relationship of $V_{CN} + V_m = 1$.

As mentioned previously [1,2], the load transfer between the nanotube and polymeric phases is less than perfect (e.g. the surface effects, strain gradients effects, intermolecular coupled stress