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# Surface energy effects on the free vibration characteristics of postbuckled third-order shear deformable nanobeams

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ABSTRACT

The prime aim of the current study is to predict the free vibration behavior of third-order shear deformable nanobeams in the vicinity of postbuckling configuration and in the presence of surface effects which includes surface elasticity, residual surface stress and surface inertia. To accomplish this end, Gurtin–Murdoch elasticity theory within the framework of third-order shear deformation beam theory is employed. In order to satisfy the balance conditions between the bulk and surfaces of nanobeam, a cubic distribution is considered for the normal stress through the thickness. By using Hamilton's principle, the non-classical governing differential equations of motion including von Karman geometric nonlinearity are derived. After using generalized differential quadrature (GDQ) method to discretize the governing equations on the basis of Chebyshev–Gauss–Lobatto grid points, the pseudo-arc length continuation technique is utilized to solve the eigenvalue problem. The natural frequencies of nanobeam corresponding to the both prebuckling and postbuckling domains are obtained for various buckling mode shapes based on the numerical solution strategy. It is demonstrated that in the prebuckling domain of the first vibration mode shape, increasing of beam thickness leads to lower natural frequency for all types of boundary conditions, but this behavior becomes reverse in the postbuckling domain.



## 1. Introduction

Due to miniaturization of electromechanical systems, nanobeams have been widely used for different applications such as strain sensors [1], optical nanocavities [2], and nano-irradiation [3]. Hence, the studies concerned with mechanical behaviors of the beams at nanoscale have been gaining much attention in nanomechanics. However, in nanostructures, the discontinuous between the sub-bodies of the system are considerable which affect the mechanical characteristics of the small-scale structures namely size effects. As a result, the predictions of the classical continuum theory for the behaviors of nanostructures are made questionable. So, various modified continuum models have been proposed and applied to remove this essential deficiency of the classical continuum theory [4–15].

One of such molecular effects is surface energy which has been clearly indicated and explained [16,17]. Because of different environment conditions, atoms at or near a free surface have different equilibrium requirements than the atoms have in the bulk of the material. This difference causes excess surface energy as the surface can be interpreted as a layer to which certain energy is attached [18]. Due to high ratio of surface area to volume in nanostructures, the effects of surface energy can be significant. Gurtin and Murdoch [19,20] developed a theoretical framework based on the continuum mechanics including surface energy effects. Based on this type of continuum elasticity theory, the surface is simulated as a mathematical membrane of zero thickness with different material properties from the underlying bulk which is completely bonded by the membrane. Herein, some of the investigations carried out about the effect of surface energy on the mechanical behaviors of nanostructures are cited.

Wang and Feng [21] and Abbasion et al. [22] studied the free vibrations of microsclae beam including surface effects based on Euler–Bernoulli and Timoshenko beam theories, respectively. Tian and Rajapakse [23] applied the Gurtin–Murdoch elasticity to take into account the surface-interface stress effects on the elastic field of an isotropic matrix with a nanoscale elliptical inhomogeneity. Lu et al. [24] used Gurtin–Murdoch elasticity theory to propose a generalized refined theory incorporating the influence of surface stress for functionally graded films. Zhao and Rajapakse [25] examined the axisymmetric solutions for an elastic layer subjected to surface

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loading including surface energy effect. Fu et al. [26] investigated the influences of surface energy on the free vibration and buckling of nanobeams in the both linear and nonlinear regimes using Galerkin's technique. Ansari and Sahmani [27] used Gurtin–Murdoch elasticity theory to predict the bending and buckling behaviors of nanobeams. They performed an analytical solution to obtain explicit formulas for critical buckling of nanobeams in the presence of surface effects. Gheshlaghi and Hasheminejad [28] examined the nonlinear flexural vibrations of simply supported Euler–Bernoulli nanobeams via an exact solution method with consideration of surface stress effect. The free vibration characteristics of rectangular nanoplates including surface stress effect were investigated by Ansari and Sahmani [29]. They implemented the Gurtin–Murdoch elasticity theory into the classical first-order shear deformation plate theory to capture size effect. Recently, Ansari et al. [30,31] predicted the postbuckling characteristics of nanobeams in the presence of surface stress by using Gurtin–Murdoch elasticity theory within the framework of Euler–Bernoulli and Timoshenko beam theories, respectively. They also investigated the surface effects on the nonlinear forced vibration response of nanobeams using surface elasticity theory [32].

A pioneer study on vibrations in the vicinity of buckled configuration of beams was conducted by Nayfeh and Emam [33]. They presented the vibrations of buckled beams around the postbuckling domain based on a closed-form solution. Rahimi et al. [34] studied recently the vibrations of functionally graded Timoshenko beams around the first buckled configuration by neglecting the in-plane inertia.

In the most cases, it is not easy to obtain exact solutions for complicated nonlinear problems, such as vibration of postbuckled beams. Therefore, proposing reliable numerical solution methodology to solve this type of problem can be an excellent approach. Motivated by this matter, the objective of the present study is to anticipate the surface effects on the nonlinear vibration response of third-order shear deformable nanobeams in the vicinity of postbuckling configuration based on a numerical solution strategy. Gurtin–Murdoch elasticity theory within the framework of third-order shear deformation beam theory is employed to develop non-classical beam model which has the capability to capture surface effects efficiently.

## 2. Preliminaries

Consider a nanobeam of length $L$, width $b$, and thickness $h$ subjected to the axial compressive load $N_{0x}$. A coordinate system $(x,y,z)$ is attached to the neutral axis of the nanobeam as the $x$-axis is taken along the length of the beam, the $y$-axis along the width, and the $z$-axis along the thickness of nanobeam. Among various types of the classical beam theory, in the third-order shear deformation beam theory, there is no shear correction factor to estimate the distribution of shear strain across beam thickness. In this theory, it is assumed that the transverse shear strain is assumed to be distributed parabolically through the beam thickness as shown in Fig. 1. According to this type of beam theory, the components of displacement vector for an arbitrary point can be defined as

$$u_x = U(x,t) + z\Psi(x,t) - \frac{4z^3}{3h^2}\left(\Psi(x,t) + \frac{\partial W(x,t)}{\partial x}\right),$$
$$u_y = 0, \quad u_z = W(x,t) \tag{1}$$

in which $U(x,t)$, $W(x,t)$ and $\Psi(x,t)$ stand for, respectively, the axial displacement of the center of sections, the lateral deflection of the beam, and the rotation angle of the cross section with respect to the vertical direction. The components of strain tensor for a third-order shear deformable nanobeam can be approximated by the von Karman relation as

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 = \frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x} - \frac{4z^3}{3h^2}\left(\frac{\partial \Psi}{\partial x} + \frac{\partial^2 W}{\partial x^2}\right) + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 \tag{2a}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = \frac{1}{2}\left(\frac{\partial W}{\partial x} + \Psi\prime\right) - \frac{2z^2}{h^2}\left(\Psi + \frac{\partial W}{\partial x}\right) \tag{2b}$$

On the basis of the linear elasticity, the non-zero stress components for the nanobeam can be introduced as

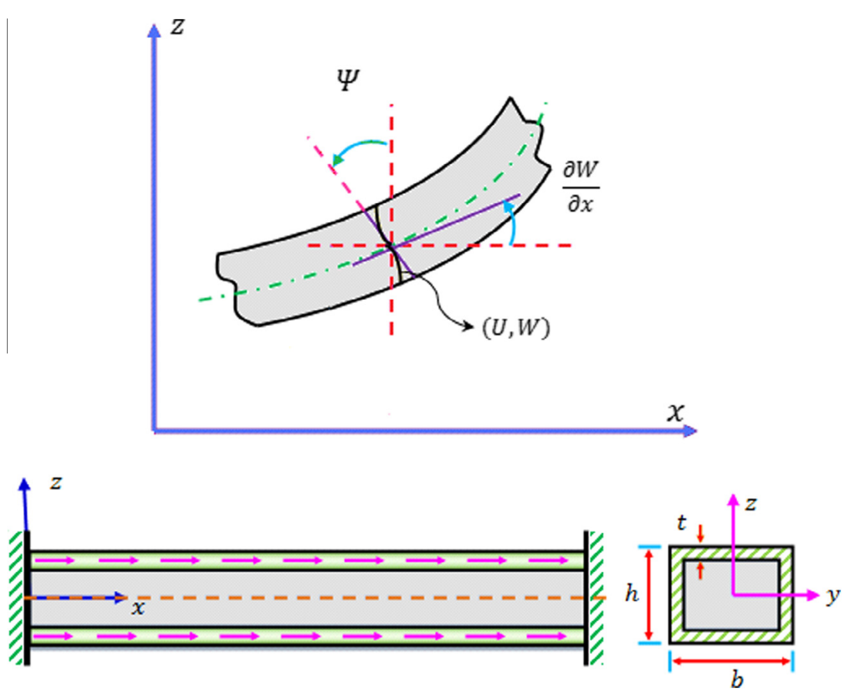


**Fig. 1.** Schematic view of a third-order shear deformable nanobeam with the kinematic parameters and coordinate system.