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Three-dimensional vibration analysis of laminated functionally graded spherical shells with general boundary conditions

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ABSTRACT

Free vibration of laminated functionally graded (FG) spherical shells with general boundary conditions and arbitrary geometric parameters is studied in this paper. The study is based on the three-dimensional shell theory of elasticity and the energy based Rayleigh–Ritz procedure. It is assumed that the material properties of the laminated FG spherical shells vary continuously through the thickness direction according to power law distribution of the volume fraction of the constituents. Under the current framework, regardless of boundary conditions, each displacement variations of the laminated FG spherical shell is invariantly expanded as a modified Fourier series in which several supplementary terms are introduced to ensure and accelerate the convergence of the expansion and all the expanded coefficients are determined by the Rayleigh–Ritz procedure. The modified Fourier series results are presented and compared with the available accurate solutions to verify the validity of the current formulation. Detailed parametric investigation is carried out to examine the influences of boundary conditions, geometric parameters and material distributions on the natural frequencies of the spherical shells. Numerous vibration results for several laminated FG spherical shells with various boundary conditions are presented for different geometric parameters and power-law exponents, which may serve as benchmark solutions for future researches to evaluate the new 2-D shell theories and to compare results obtained by approximate numerical methods.

1. Introduction

The outstanding properties of advanced laminated composite structures, such as high strength-to-weight and stiffness-to-weight ratios, have led to their widespread use in space aircrafts, areas of civil and deep-sea equipments. However, in conventional laminated composite structures, different homogenous or composite laminae are bonded together to obtain enhanced mechanical and thermal properties, which can result in interlaminar stress concentration and leading to debonding failure due to the abrupt change in material properties across the interface. One way to overcome this drawback is to use functionally graded materials (FGMs). The FGMs are a new class of advanced composite materials first advocated by a group of researchers in Japan [1,2], which have continuous variation of material properties through their thickness, and thus eliminating the interface problems.

Recently, FGMs structures are extensively applied in various fields and are of great interest for engineers and researchers [3–21]. As a kind of important structural elements, in recent decades, laminated spherical shells made from FGMs are applied in an increasing number of engineering structures to satisfy special functional requirements. Understanding the vibration characteristics of these structural elements is particularly important for engineers to design suitable structures with low vibration and noise radiation characteristics. However, despite the great efforts devoted to vibration analysis of homogeneous and composite laminated spherical shells, the developing of an accurate and reliable method for vibrations of laminated FGM spherical shells with general boundary conditions remains a challenge and is the focus of the present work.

A spherical shell is a doubly-curved shell with constant curvature in the meridional and circumferential directions and the two radii of curvature are equal. It is noticeable that the spherical shells are very stiff for both in-plane and bending loads due to the curvature of the middle surface, which is also a reason for the analysis difficulties of these shells, especially the exact three-dimensional elasticity (3-D) analysis. Usually, researchers eliminate the shell thickness dimension by introducing a few assumptions for approximation and simplify the 3-D shell problems to various

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two-dimensional (2-D) representations. Mainly, three major representations, i.e., the classical shell theories (CSTs), the first-order shear deformation theories (FSDTs) and the higher-order shear deformation theories (HSDTs) have been developed. The CSTs (e.g., Donnell's, Love's, Flügge's shell theories, etc.) which are obtained by neglecting the in-plane inertia, transverse shear deformation and rotary inertia are only adequate to thin shells in the lower-frequency range. The FSDTs provide a better prediction of the vibration results of slightly thick shells than the classical ones by considering the transverse shear deformation of shells. However, the transverse shear strains in FSDTs are assumed to be constant across the thickness. Therefore shear correction factors have to be incorporated to adjust the transverse shear stiffness in the actual calculation. The shear correction factors, which depend not only on the geometric parameters, but also on the loading and boundary conditions, are difficult to determine [22]. One way to overcome this difficulty is to use the HSDTs. In the HSDTs, the displacement field accommodates quadratic variation of transverse shear strains and the transverse shear stresses vanish on the top and bottom. Thus there is no need to use shear correction factors. The HSDTs are better than the FSDTs for moderately thick shells but are often incapable of accurately describing the state of stress and strain of thick shells since the transverse normal stress and strain components are ignored [22]. Moreover, the spherical shells in engineering practices may be of recognizable thickness-length or thickness-radius ratios. The conventional 2-D shell theories are limited in solving the whole spectrum of thick shell problems [5]. Recently, based on the Carrera Unified Formulation (CUF), Tornabene et al. [23,24] developed a two dimensional General Higher-order Equivalent Single Layer (GHESL) approach to investigate the static and vibration behaviors of doubly-curved laminated composite shells and panels. In this approach, the degree of freedom expansion is related to both the in-plane displacements and the thickness direction, and the zig-zag effect has been considered, therefore, it is applicable to obtain the whole natural frequencies of moderately thick and thick shells. Besides the GHESL approach and other advanced 2-D theories [25], the 3-D theories which accounts for all the transverse stress and strain components may be the best choice.

In recent decades, several researches concerned to vibration analysis of spherical shells have been carried out based on the aforementioned 2-D and 3-D shell theories. In order to properly focus on the emphasis of the present work, some research papers and articles on this topic are given in the following. Ganapathi [3] studied the dynamic stability behavior of a clamped functionally graded materials spherical shell structural element subjected to external pressure load by using the Newmark's integration technique and the effective material properties are evaluated using a homogenization method. Qu et al. [4–6] combined a modified variational principle with a multi-segment partitioning procedure to investigate the vibration characteristics of spherical shells and bodies. An exact closed-form analysis method was developed by Fadaee et al. [7] for calculating the natural vibrations of FG moderately thick spherical panels using Lévy-type boundary conditions. The FSDT and GDQ method was used by Tornabene and Viola [8,9] to study the dynamic behavior of functionally graded parabolic and circular panels and shells of revolution. Pradyumna and Bandyopadhyay [10] carried out free vibration solutions of functionally graded curved panels using a higher-order formulation. Wu and Tsai [11] presented the three-dimensional (3D) solution for the static analysis of functionally graded annular spherical shells by using the methods of differential quadrature and asymptotic expansion. Su et al. [13] analyzed the vibrations of moderately thick spherical caps with arbitrary boundary conditions based on the FSDT. Some other contributors are given in Refs. [26–43]. The development of researches on this subject has been well documented in several monographs respectively by Reddy [22], Carrera et al. [25], Qatu [44] and reviews [45–48].

The above review reveals that vibrations of FG spherical shells are of great interest for researchers. However, most of the previous studies are confined to particular spherical shells based on 2-D shell theories. And most of the available solution procedures in the open literature are often only customized for a limited set of boundary conditions, e.g., a FG spherical shell with free, simply-supported, or clamped boundary conditions, which may not appropriate for practical application. The only works are available in the open literature is that of Qu and Meng [5,6] contributed a modified variational formulation for FG spherical shell with arbitrary boundary conditions. To the best of the authors' knowledge, there is no literature for 3-D vibration analysis of laminated FG spherical shells with arbitrary geometric dimensions and general boundary conditions and the authors attempt to fill this apparent void.

The main objective of this paper is to develop a unified, accurate and reliable method for free vibration of thick laminated FG spherical shells with general boundary conditions and arbitrary geometric parameters. For this purpose, the three-dimensional shell theory of elasticity and the energy based Rayleigh–Ritz procedure are employed in the theoretical formulation. The material properties of the FGM layers are assumed to vary continuously through the thickness direction according to power law distribution of the volume fraction of the constituents. The general boundary conditions of a shell are implemented by introducing three groups of linear springs which are continuously distributed along the boundary and the given boundary conditions of the shell can be readily achieved by assigning these springs at proper stiffness. Regardless of boundary conditions, each displacement variations of a laminated FG spherical shell is invariantly expanded as a modified Fourier series in which several supplementary terms are introduced to ensure and accelerate the convergence of the expansion. A systematic comparison including isotropic and FG spherical shells subjected to various boundary conditions between the present results and those available in the open literature as well as solutions provided by FEM analyses is carried out to validate the convergence, accuracy and reliability of the proposed method, with excellent agreements obtained. Comprehensive studies on the effects of boundary conditions, geometric parameters and material distributions are also reported. Numerous new results for six types of laminated FG spherical shells with various boundary conditions are tabulated for several thickness-radius ratios, power-law exponents and different geometric parameters, which may serve as benchmark solution for future researches.

2. Theoretical formulations

2.1. The model

Consider a thick laminated FG spherical shell with inner radius R_0, mean radius R, outer radius R_1 and uniform thickness h as shown in Fig. 1(a). The cross-section of the spherical shell and the coordinate system are given in the figure. The ends of the shell (top and bottom) are determined by ϕ_0 and ϕ_1 (where $\Delta\phi = \phi_1 - \phi_0$) and the axes ϕ, r and θ of the coordinate system are taken in the axial, circumferential and radial directions of the shell, respectively. For the sake of brevity, all layers of the laminated FG spherical shell have the same thickness and made from one of the following materials: FGM$_I$, FGM$_{II}$ or metal. The FGM$_I$ and FGM$_{II}$ layers are made from a mixture of ceramic and metal by gradually changing the volume fraction of the ceramic (V_c) in the thickness direction and the continuous material properties of them can be expressed as [8,13]: