Composite Structures 116 (2014) 589–594

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A novel method for the partition of mixed-mode fractures in 2*D* elastic laminated unidirectional composite beams

C.M. Harvey, J.D. Wood, S. Wang *, A. Watson

Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, United Kingdom

ARTICLE INFO

Article history:
Available online 9 June 2014

Keywords:
Energy release rate
Fracture mode partitioning
Laminated unidirectional composites
Mixed-mode fracture
Orthogonal pure modes

ABSTRACT

A powerful method for partitioning mixed-mode fractures on rigid interfaces in laminated unidirectional double cantilever beams (DCBs) is developed by taking 2*D* elasticity into consideration in a novel way. Pure modes based on 2*D* elasticity are obtained by introducing correction factors into the beam-theory-based mechanical conditions. These 2*D*-elasticity-based pure modes are then used to derive a 2*D*-elasticity-based partition theory for mixed-mode fractures. Excellent agreement is observed between the present partition theory and Suo and Hutchinson's partition theory (Suo and Hutchinson, 1990) [1]. Furthermore, the method that is developed in this work has a stronger capability for solving more complex mixed-mode partition problems, for example, in the bimaterial case (Suo and Hutchinson, 1990) [1].

1. Introduction

The present work revisits the partitioning of mixed-mode fractures in laminated unidirectional composite double cantilever beams (DCBs) with rigid interfaces by taking 2*D* elasticity into consideration in a novel way. In Suo and Hutchinson's work [1], conventional 2*D* elasticity theory is employed in conjunction with stress intensity factors in order to give accurate partitions. This conventional approach however often has limitations in dealing with more complex problems, for example, in the bimaterial case where the partition relies on extensively tabulated numerical results over a finite range of geometries and material configurations [1]. The present work aims to develop a novel and powerful method to calculate energy release rate (ERR) partitions with the same level of accuracy as in Suo and Hutchinson's work [1]. Furthermore, it aims for the method to have a stronger capability for solving more complex mixed-mode partition problems (like the bimaterial one described above) than the conventional method in Ref. [1] has. The structure of the paper is as follows. The novel method is developed in Section 2. Comparisons with several existing partition theories are presented in Section 3. In particular, these comparisons include ones against Suo and Hutchinson's partition theory [1] since it is regarded as the most accurate. Conclusions are made in Section 4.

2. Development of the novel method

Fig. 1a shows a laminated unidirectional composite DCB with its geometry and tip bending moments M_1 and M_2, and axial forces N_1 and N_2. The crack influence zone extends to a point A, a Δa-distance ahead of the crack tip B. Fig. 1b only shows the sign convention of the interface normal stress σ_n and shear stress τ_s instead of any representative distribution. Beyond point A, both the normal stress σ_n and shear stress τ_s becomes zero.

Based on the authors' previous work [2–5], the total ERR G is calculated as follows:

$$G = \frac{1}{2\bar{E}b}\left[\frac{M_{1B}^2}{I_1} + \frac{M_{2B}^2}{I_2} - \frac{1}{I}\left(M_{1B} + M_{2B} - \frac{h_2 N_{1Be}}{2}\right)^2 + \left(\frac{1}{A_1} - \frac{1}{A}\right)N_{1Be}^2\right]$$
$$= \{\, M_{1B} \quad M_{2B} \quad N_{1Be} \,\}[C]\{\, M_{1B} \quad M_{2B} \quad N_{1Be} \,\}^T \tag{1}$$

where $N_{1Be} = N_{1B} - N_{2B}/\gamma$ with $\gamma = h_2/h_1$, b is the width of the beam, and $\bar{E}$ is the effective axial Young's modulus for orthotropic material; for isotropic material then $\bar{E} = E/(1-\nu^2)$ for plane strain and $\bar{E} = E$ for plane stress where E is the Young's modulus and ν the Poisson's ratio [1]. M_{1B} and M_{2B} are the two bending moments at the crack tip B, and N_{1B} and N_{2B} are the axial forces at the crack tip B. Other symbols have their conventional meanings. G is of quadratic form in terms of M_{1B}, M_{2B} and N_{1Be} with the coefficient matrix $[C]$, which is given in full in the Appendix A. The total ERR is the same for both the Euler and Timoshenko beam theories and for 2*D* elasticity theory. The mode I and II partitions of ERR are however different. Approximate 2*D* partition theories have been given in

* Corresponding author. Tel.: +44 1509227252.
E-mail addresses: c.m.harvey@lboro.ac.uk (C.M. Harvey), J.Wood@lboro.ac.uk (J.D. Wood), s.wang@lboro.ac.uk (S. Wang), A.Watson@lboro.ac.uk (A. Watson).

http://dx.doi.org/10.1016/j.compstruct.2014.05.041
0263-8223/

Nomenclature

a	crack length in a DCB
A_1, A_2, A	cross section areas of upper, lower and intact beams
b	width of a DCB
c_θ, c_β	2*D* elasticity correction factors for θ mode I and β mode II
E	Young's modulus
G, G_I, G_{II}	total, mode I and II ERRs
G_θ, G_β	θ mode I and β mode II ERRs
h_1, h_2, h	thicknesses of upper, lower and intact beams
I_1, I_2, I	second moments of upper, lower and intact beams
L	length of a DCB
M_1, M_2	DCB tip bending moments on upper and lower beams
M_{1B}, M_{2B}	crack tip bending moments on upper and lower beams
N_1, N_2	DCB tip axial forces on upper and lower beams
N_{1B}, N_{2B}	crack tip axial forces on upper and lower beams
N_{1Be}	crack tip effective axial force on upper beam
γ	thickness ratio
(θ, β)	zero shearing displacement, zero opening force orthogonal pure modes pair
(θ', β')	zero shearing force, zero opening displacement orthogonal pure modes pair
ν	Poisson's ratio
σ_n, τ_s	interface normal and shear stresses
Δa	crack influence length in a DCB

Abbreviations

DCB	double cantilever beam
ERR	energy release rate

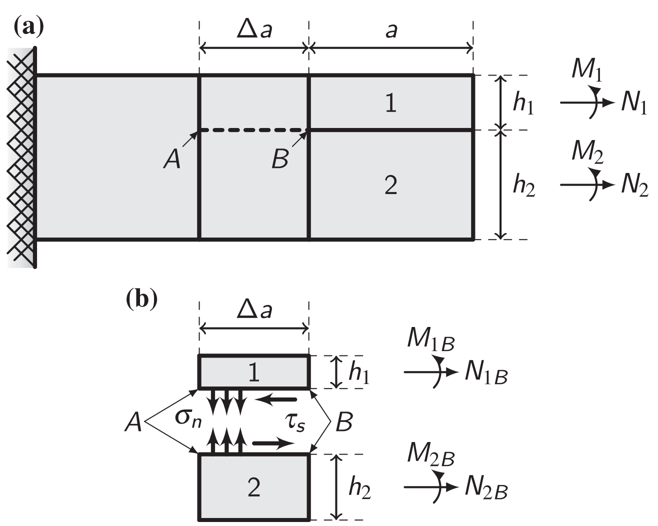

Fig. 1. A laminated unidirectional composite DCB. (a) General description. (b) Details of the Δa-length crack influence region.

Refs. [2–4]. Here, a partition theory of the same level of accuracy as that of the work in Ref. [1] is obtained by developing a novel and powerful method.

By using the same hypothesis as in Refs. [2–4,6], namely that there generally exist two sets of orthogonal pure modes for rigid interface fracture in DCBs, the total ERR G in Eq. (1) can be partitioned as

$$G_I = c_I\left(M_{1B} - \frac{M_{2B}}{\beta_{1-2D}} - \frac{N_{1Be}}{\beta_{2-2D}}\right)\left(M_{1B} - \frac{M_{2B}}{\beta'_{1-2D}} - \frac{N_{1Be}}{\beta'_{2-2D}}\right) \tag{2}$$

$$G_{II} = c_{II}\left(M_{1B} - \frac{M_{2B}}{\theta_{1-2D}} - \frac{N_{1Be}}{\theta_{2-2D}}\right)\left(M_{1B} - \frac{M_{2B}}{\theta'_{1-2D}} - \frac{N_{1Be}}{\theta'_{2-2D}}\right) \tag{3}$$

where c_I and c_{II} are two constants, and $(\theta_{i\text{-}2D}, \beta_{i\text{-}2D})$ and $(\theta'_{i-2D}, \beta'_{i-2D})$ with $i = 1, 2$ represent the first and second sets orthogonal pure modes respectively. The subscript 2*D* denotes that the pure modes are based on 2*D* elasticity theory. For example, when $M_{2B} = \theta_{1\text{-}2D}M_{1B}$ and $N_{1Be} = 0$, the pure mode I mode occurs as the relative shearing displacement just behind the crack tip is zero. This pure mode I is denoted by $\theta_{1\text{-}2D}$. Its orthogonal pure mode II is $\beta_{1\text{-}2D}$ which corresponds to zero crack tip opening force. Here, the 'orthogonal' means

$$\{1 \quad \theta_{1-2D} \quad 0\}[C]\{1 \quad \beta_{1-2D} \quad 0\}^T = 0 \tag{4}$$

For simplicity, Eq. (4) can be written as $\theta_{1\text{-}2D} = \text{orthogonal}(\beta_{1\text{-}2D})$. Similarly, when $M_{2B} = \theta'_{1-2D}M_{1B}$ and $N_{1Be} = 0$, the pure mode I mode occurs as the crack tip shearing force is zero. This pure mode I is denoted by θ'_{1-2D}. Its orthogonal pure mode II is β'_{1-2D} which corresponds to zero crack tip opening displacement.

The work in Refs. [2–4,6] has shown that in Euler beam theory with rigid interfaces, the two sets of orthogonal pure modes do not coincide and that this results in 'stealthy' interactions which change the ERR partitions G_I and G_{II} but do not change the total ERR G. The work in Refs. [2–6] also shows that in Timoshenko beam theory with either rigid or non-rigid interfaces, these two sets of modes coincide on the first set of pure modes from Euler beam theory resulting in no stealthy interaction. Furthermore, Ref. [1] shows that the two sets also coincide in 2*D* elasticity theory for rigid interfaces, i.e. $(\theta_{i-2D}, \beta_{i-2D}) = (\theta'_{i-2D}, \beta'_{i-2D})$ with $i = 1, 2$. Therefore, Eqs (2) and (3) become here for laminated unidirectional composite DCBs with rigid interfaces in 2*D* elasticity,

$$G_I = c_I\left(M_{1B} - \frac{M_{2B}}{\beta_{1-2D}} - \frac{N_{1Be}}{\beta_{2-2D}}\right)^2 \tag{5}$$

$$G_{II} = c_{II}\left(M_{1B} - \frac{M_{2B}}{\theta_{1-2D}} - \frac{N_{1Be}}{\theta_{2-2D}}\right)^2 \tag{6}$$

where

$$c_I = G_{\theta_{1-2D}}\left(1 - \frac{\theta_{1-2D}}{\beta_{1-2D}}\right)^{-2}, \quad c_{II} = G_{\beta_{1-2D}}\left(1 - \frac{\beta_{1-2D}}{\theta_{1-2D}}\right)^{-2} \tag{7}$$

$$G_{\theta_{1-2D}} = \frac{1}{2bE}\left(\frac{1}{I_1} + \frac{\theta_{1-2D}^2}{I_2} - \frac{(1+\theta_{1-2D})^2}{I}\right),$$
$$G_{\beta_{1-2D}} = \frac{1}{2bE}\left(\frac{1}{I_1} + \frac{\beta_{1-2D}^2}{I_2} - \frac{(1+\beta_{1-2D})^2}{I}\right) \tag{8}$$

Now, the key task is to determine the orthogonal pure mode set $(\theta_{i\text{-}2D}, \beta_{i\text{-}2D})$ with $i = 1, 2$. At this point, it is important to note that the orthogonal property demonstrated in Eq. (4) exists between any pair of pure modes in the pure mode set $(\theta_{i\text{-}2D}, \beta_{i\text{-}2D})$ with $i = 1, 2$. That is, $\theta_{1\text{-}2D} = \text{orthogonal}(\beta_{1\text{-}2D}$ and $\beta_{2\text{-}2D})$ and $\theta_{2\text{-}2D} = \text{orthogonal}(\beta_{1\text{-}2D}$ and $\beta_{2\text{-}2D})$. As long as one pure mode is known, say $\theta_{1\text{-}2D}$, the others can be obtained by using the orthogonal property. This knowledge provides a powerful methodology to find all the pure modes and to partition mixed modes, which will be used in the following development. It is seen now that the central task of the present work is to determine $\theta_{1\text{-}2D}$. In what follows, a novel method is developed for this task.