Composite Structures 116 (2014) 595–611

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Size-dependent generalized thermoelasticity model for Timoshenko micro-beams based on strain gradient and non-Fourier heat conduction theories

Ehsan Taati [a], Masoud Molaei Najafabadi [b,*], J.N. Reddy [c]

[a] Mechanical Engineering Department, Sharif University of Technology, Azadi Ave., Tehran, Iran
[b] Mechanical Engineering Department, Amirkabir University of Technology, No. 424, Hafez Ave., PO Box 15875-4413, Tehran, Iran
[c] Mechanical Engineering Department, Texas A&M University, College Station, TX 77843-3123, USA

ARTICLE INFO

Article history:
Available online 11 June 2014

Keywords:
Size dependent model
Coupled thermoelasticity
Non-Fourier heat conduction model
Strain gradient theory
Timoshenko micro-beam

ABSTRACT

The governing equations of coupled thermoelasticity of Timoshenko micro-beams are developed based on the generalized thermoelastic theory and non-Fourier heat conduction model. Such problems may arise in MEMS such as micro-pumps as well as micro-sensors. The present model is on the basis of non-classical continuum theory and non-Fourier heat conduction model which has capability of capturing the size-effect in micro-scaled structures. Governing equations and both classical and non-classical boundary conditions of motion are obtained using the variational approach. As the case study, the present model is utilized for the simply supported micro-beams subjected to a constant impulsive force per unit length. It is assumed the ends of micro-beam are held at a constant temperature. Comparison of the results with those obtained by the modified couple stress and the classical continuum theories is carried out. The illustration of the existence of thermal damping in the coupled thermoelastic problem is another important contribution of this study. Findings indicate that the transient response is extremely sensitive to the material length scale parameters. In addition, only the proposed size dependent thermoelastic model can predict the unstable transient response of the micro beams under thermomechanical shocks.

1. Introduction

Beams and plates have been broadly applied in micro and nano-scale devices and systems such as biosensors, MEMS, as well as NEMS [1–4]. In these applications, since the thickness of beams and plates are clearly in the order of microns and sub-microns, the size-dependent deformation and vibration response of micro-scale structures have been experimentally observed [e.g. Refs. 5,6]. Due to the lack of the material length scale parameter in the classical constitutive equations, the classical continuum mechanic theories are not capable of describing the size-effects that are observed in micro-scale structures. Therefore, the beam (and plate) models based on the classical elasticity theory cannot accurately predict the mechanical behavior of nano devices made of such elements. In order to capture the size effect in micro-structures, some non-classical continuum theories, such as the nonlocal, strain gradient, and couple stress have been introduced and developed to investigate the micro-scale structures.

In a nonlocal theory one assumes that the stress at a point is a function of strains at all points in the continuum. In such theories the internal length scale is introduced into the constitutive equations as a material parameter. This idea was initiated in the work of Eringen [7]. There appeared numerous papers based on this idea, mostly dealing with beams and some with plates [see e.g., Refs. 8–11]. In 1960s, the couple stress elasticity theory as a non-classic theory was introduced by some researchers [12]. This theory is capable to interpret the size effects with in view of two higher-order material constants in the corresponding constitutive equations. By simplifying on the complicated relations of couple stress theory, Yang et al. [13] presented the modified couple stress theory that was able to explore the size effects using a single material length scale parameter. Kong et al. [14] analyzed the size-dependent natural frequency of Euler–Bernoulli micro-beams. They derived the governing equations and boundary conditions for an Euler–Bernoulli beam via the modified coupled stress theory using the Hamilton principle.

Among the non-classical-continuum theories, Mindlin [15] proposed the strain gradient elasticity that is the most complex form of higher order continuum theory. The first strain gradient theory

* Corresponding author. Tel.: +98 2166165511.
E-mail address: mmolaei@aut.ac.ir (M. Molaei Najafabadi).

http://dx.doi.org/10.1016/j.compstruct.2014.05.040
0263-8223/

was introduced by Mindlin and Eshel [16]. The numbers of five supplementary parameters along with the classical Lame constants are employed in this theory. By applying some modifications in the Mindlin theory, the simple form of the strain gradient theory proposed by Lam et al. [6] was successfully used to predict the size effects for small scale structures [17,18]. In this theory, three material length scale parameters are introduced to characterize the dilatation gradient tensor, the deviatoric stretch gradient tensor, and the symmetric rotation gradient tensor, respectively. Also, this theory can be reduced in a special case to the modified couple stress theory by ignoring the dilatation and deviatoric stretch gradient.

There are different theories relevant to heat conduction in solids. The classic parabolic heat conduction theory is based on the Fourier law as the constitutive relation. Fourier law implicitly postulates that the propagation speed of thermal disturbances is infinite which is in contrast to what occurs in reality. Although this law gives quite reliable results in many practical situations, but it breaks down for materials in situations involving extreme temperature gradients, low temperatures and conduction at very short periods of time such as pulsed-laser processing of metals and semiconductors, film applications, laser surgery, etc., where the typical response time is in the order of picoseconds and consequently the wave nature of thermal propagation becomes dominant. In addition, Fourier law is ineffective at the very small length scales associated with small-scale systems (e.g., nano- or micro-scale systems). In these applications the results predicted by using the Fourier heat conduction model differ significantly from the experimental results. In the non-Fourier heat conduction model, the thermal relaxation time is used to alleviate the shortcomings of the Fourier law associated with small scale systems.

In recent years, many researchers utilized the non-classical continuum theories to formulate and investigate the size-dependent mechanical behavior of the beams and plates. In this regard, Park and Gao [19] presented an Euler– Bernoulli beam model based on the modified couple stress theory and investigated the effect of material length scale parameter on the static mechanical properties of micro-beam. Ma et al. [20] investigated the size-dependent static behavior of a homogeneous Timoshenko beam model based on the modified couple stress theory. Kong et al. [21] developed a size dependent beam model for static and dynamic analysis of the micro beams using strain gradient theory. Furthermore, Wang et al. [22] presented a micro scale Timoshenko beam model based on strain gradient theory. They derived simultaneously the governing equations, initial conditions and boundary conditions using the Hamilton principle. Their model incorporated with Poisson effect, contains three material length scale parameters and can consequently capture the size effect. Ashoori and Mahmoodi [23] proposed a Kirchhoff micro-plate model based on the modified strain gradient elasticity theory to capture size effects. They showed that the method provides accurate predictions with a very fast convergence. Also, the obtained numerical results revealed when the plate thickness is small and comparable to the material length scale parameters, the difference between the deflections predicted by the modified strain gradient model, the couple stress model and the classical model becomes high. In recent years, a number of papers dealing with the modified couple stress theory as applied to (including functionally graded) shear deformable beams and plates have appeared in the literature [24–30]. Research on thermoelastic vibration of beams has been widely progressed in the previous studies. In this regard, Landau and Lifshitz [31] presented an exact expression for the attenuation coefficient of thermoelastic vibration. However, they did not express a rigorous derivation and solution of the governing equations. Manolis and Beskos [32] surveyed the effects of damping and axial loads on the vibration of beams subjected to fast heating on the surface. They ignored the coupling effect between the stress and temperature fields. Massalas and Kalpakidis [33] investigated coupled thermoelastic vibration of a simply supported beam. They presented an analytical solution for the coupled thermoelasticity of beams made of homogeneous and isotropic materials with the assumptions of the Euler–Bernoulli and Timoshenko beam models, respectively. Givoli and Rand [34] analyzed the effect of thermoelastic coupling on dynamic behavior of a rod. They demonstrated the nature of the dynamic response of the rod is varied considerably when the frequency of the thermal loading is nearby to the critical frequency of the structure. Rezazadeh et al. [35] investigated the thermoelastic damping in a capacitive micro-beam resonator using hyperbolic heat conduction model. Furthermore, Vahdat and Rezazadeh [36] studied the effect of axial and residual stresses on thermoelastic damping in capacitive micro-beam resonator.

Till to date, only a few studies relative to the size effect on the coupled thermoelastic of micro-beam have been reported. Guo and Rogerson [37] studied the size effect on thermoelastic coupling in a doubly clamped elastic prism beam to examine its size-dependence. Sun et al. [38] developed the formulation for coupled thermoelastic of micro-beam using non-Fourier heat conduction model. They illustrated the size-effect induced by thermoelastic coupling would disappear when the thickness of the micro-beam is over a critical value that depends on the material properties and the boundary conditions. Rezazadeh et al. [39] presented analytical expressions for the quality factor of thermoelastic damping by applying modified couple stress theory for plane stress and strain conditions. They demonstrated while the beam thickness is close to the length-scale parameter of the material, the modified couple stress theory diverges from the classical theory; otherwise, the two theories converge to each other. Taati et al. [40] combined modified couple stresses and non-Fourier heat conduction to capture size effects in the microscale. They indicated that while the non-Fourier heat conduction model is employed, the modified couple stress theory predicts larger deflection compared with the classical theory.

Considering the aforementioned studies, it can be concluded that, mostly, the governing equations of dynamic motion have been derived based on the classical continuum mechanics theory (with the exception of Reddy [25] and Reddy and Kim [27], who derived equations of motion of functionally graded beams and plates with modified couple stress theory). Furthermore, the classical Fourier heat conduction model has been utilized to derive the thermoelastic coupling equation, assuming the infinite speed of the heat propagation. Moreover, there is no study for developing a combined model for coupled thermoelasticity of micro-beams based on the strain gradient and non-Fourier heat conduction theories. This study was undertaken to fill the gap in the literature by deriving the governing equations and boundary conditions for coupled thermoelasticity of Timoshenko micro-beams based on the strain gradient and non-Fourier heat conduction theories. A size dependent model using the thermal relaxation time for transient analysis of coupled thermoelasticity of Timoshenko micro-beams is presented. In addition, the size effects on the transient response of the simply supported micro-beam subjected to constant impulsive force per unit length, where two ends of the beam are held at a constant temperature of the environment, are discussed.

2. Preliminaries

On the basis of the non-classical continuum theory, Lam et al. [6] presented the strain gradient theory in which added to the classical equilibrium equations of forces and moments, a new additional equilibrium equation is considered to govern the behavior of higher-order stresses. They demonstrated that the total deformation energy density becomes a function of the symmetric strain