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Thermo-electro-mechanical vibration of size-dependent piezoelectric cylindrical nanoshells under various boundary conditions

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ABSTRACT

Thermo-electro-mechanical vibration of piezoelectric cylindrical nanoshells is studied using the nonlocal theory and Love's thin shell theory. The governing equations and boundary conditions are derived using Hamilton's principle. An analytical solution is first given for the simply supported piezoelectric nanoshell by representing displacement components in the double Fourier series. Then, the differential quadrature (DQ) method is employed to obtain numerical solutions of piezoelectric nanoshells under various boundary conditions. The influence of the nonlocal parameter, temperature rise, external electric voltage, radius-to-thickness ratio and length-to-radius ratio on natural frequencies of piezoelectric nanoshells are discussed in detail. It is found that the nonlocal effect and thermoelectric loading have a significant effect on natural frequencies of piezoelectric nanoshells.

1. Introduction

Due to the pioneering work by Wang and his co-authors [1], piezoelectric nanomaterials (e.g. ZnO, ZnS, PZT, GaN, BaTiO3, etc.) and their nanostructures (e.g. nanowires, nanobelts, nanorings, nanohelices, nanosprings, etc.) have received the attention of many researchers. Extensive technical literature on the subject can be found in the review article by Fang et al. [2]. Piezoelectric nanostructures have been regarded as the next-generation piezoelectric materials because of their inherent nanosized piezoelectricity. They exhibit an enhanced piezoelectric effect, novel electrical, mechanical, physical, and chemical properties, and the coupling between piezoelectric and semiconducting properties [2]. These distinct features make them suitable for potential applications in many nanodevices, such as nanoresonators [3], field effect transistors [4], light emitting diodes [5], chemical sensors [6], and nanogenerators [7].

Piezoelectric nanostructures have the dimension varying from several hundred nanometers to just a few nanometers. On this scale, the size effects become very important. The size-dependent material properties of piezoelectric nanostructures have been observed in both experiments and atomistic simulations [8–10]. Therefore, the size effect should be taken into account in theoretical and experimental studies of piezoelectric nanostructures. It should be pointed out that Eringen' nonlocal theory [11–13] has been widely accepted and applied to analyze the size effect on nanostructures. Based on this theory, the bending [14,15], buckling [14–17], linear vibration [14,15,18,19], nonlinear vibration [20–22], postbuckling [23,24] and wave propagation [25–27] problems of carbon nanotubes, graphene sheets, mass sensors, nanowires, and so on, have been extensively studied using the nonlocal nanobeam model, nonlocal nanoplate model, and nonlocal nanoshell model. For more details, refer to the review articles by Wang and Li [28] and Arash and Wang [29].

The investigations cited above were mainly concerned with the nonlocal effect on elastic nanostructures, such as carbon nanotubes and graphene sheets. Recently, Ke and Wang [30] and Ke et al. [31] extended the nonlocal theory to piezoelectric nanostructures. They analyzed the thermo-electro -mechanical linear and nonlinear vibration of piezoelectric nanobeams based on the nonlocal Timoshenko beam theory. Further, Liu et al. [32] studied the vibration of piezoelectric nanoplates based on the nonlocal Kirchhoff plate theory. The nanoplate was subjected to a biaxial force, an applied voltage and a uniform temperature rise. Wang et al. [33] examined the influences of both surface and small scale effects on the bending behavior of a piezoelectric nanowire by using the beam model, surface elasticity theory and nonlocal theory. Also, Hosseini-Hashemi et al. [34] studied free vibration of functionally graded piezoelectric nanobeams by considering the surface effect (including surface elasticity, surface stress, and surface density) as well as the nonlocal effect. In particular, Arani and his co-workers presented

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comprehensive investigations on the size-dependent behavior of Boron Nitride nanotubes (BNNTs) using the nonlocal piezoelectric Timoshenko beam theory [35,36] and shell theory [37,38]. Studies based on the nonlocal theory might be helpful for understanding the size-dependent electromechanical properties of piezoelectric nanowires, nanofilms, and nanoshells, and they are important for the design of piezoelectric nanodevices.

In this paper, we present a thermo-electro-mechanical vibration of piezoelectric cylindrical nanoshells based on the nonlocal theory and Love's thin shell theory. The nonlocal nanoshell model is developed to capture the size effect in piezoelectric nanostructures. Hamilton's principle is employed to derive the governing equations and boundary conditions. An analytical solution is first given for the simply supported piezoelectric nanoshell by using Navier's solution method. Then, the differential quadrature (DQ) method is used to obtain numerical solutions for vibration of piezoelectric nanoshells with different types of end supports. A detailed parametric study is conducted to highlight the influences of the nonlocal parameter, temperature rise, external electric voltage, radius-to-thickness ratio, and length-to-radius ratio on natural frequencies of piezoelectric nanoshells.

The novelty of this paper is threefold: (1) a piezoelectric nanoshell model is developed to incorporate the effects of the nonlocal parameter, temperature rise, and external electric voltage; (2) the electric potential of the piezoelectric nanoshell is assumed as a combination of a cosine and linear variation; and (3) both analytical and numerical results are presented for the vibration of piezoelectric nanoshells.

2. Nonlocal theory for piezoelectric materials

In the Eringen's nonlocal elasticity theory [11–15], the stress at a reference point in a body depends not only on the strain at that point but also on all points of the body. In the nonlocal piezoelectric theory, the stress and electric displacement at a reference point depends not only on the strain and electric field at that point but also at all other points of the body. Mathematically, the basic equations for a homogeneous and nonlocal piezoelectric solid with zero body force can be written as [30,31,39,40]

$$\sigma_{ij} = \int_V \alpha(|x'-x|,\tau)\left[C_{ijkl}\varepsilon_{kl}(x') - e_{kij}E_k(x') - \lambda_{ij}\Delta T\right]\mathrm{d}x', \tag{1}$$

$$D_i = \int_V \alpha(|x'-x|,\tau)[e_{ikl}\varepsilon_{kl}(x') + \in_{ik}E_k(x') + p_i\Delta T]\mathrm{d}x', \tag{2}$$

$$\sigma_{ij,j} = \rho\ddot{u}_i, \quad D_{i,i} = 0, \tag{3}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\tilde{\Phi}_{,i} \tag{4}$$

where $i,j,k,l = 1,2,3$; σ_{ij}, ε_{ij}, D_i, E_i and u_i are the stress, strain, electric displacement, electric field and displacement components, respectively; C_{ijkl}, e_{kij}, $\in_{ik}$, λ_{ij}, p_i and ρ are the components of the elasticity tensor, piezoelectric tensor, dielectric tensor, thermal modulus tensor, and pyroelectric vector, mass density, respectively; ΔT is the temperature rise; $\tilde{\Phi}$ is the electric potential; $\alpha(|x'-x|,\tau)$ is the nonlocal attenuation function; $|x'-x|$ is the Euclidean distance; $\tau = e_0a/l$ is the scale parameter, where e_0 is a material constant which is determined experimentally or approximated by matching the dispersion curves of the plane waves with those of the atomic lattice dynamics; and a and l are the internal and external characteristic lengths of the nanostructures, respectively.

According to Eringen [12], it is possible to represent the integral constitutive relations (1) and (2) in an equivalent differential form as

$$\sigma_{ij} - (e_0a)^2\nabla^2\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \lambda_{ij}\Delta T, \tag{5}$$

$$D_i - (e_0a)^2\nabla^2 D_i = e_{ikl}\varepsilon_{kl} + \in_{ik}E_k + p_i\Delta T, \tag{6}$$

where ∇^2 is the Laplace operator; e_0a is the scale coefficient revealing the size effect on the response of nanostructures. In Eqs. (1)–(6) comma followed by a subscript (k) denotes differentiation with respect to x_k.

3. Nonlocal piezoelectric cylindrical nanoshell model

Consider a piezoelectric cylindrical nanoshell with the length L, radius R, and thickness h, and subjected to an applied electric voltage $\tilde{\Phi}(x,\theta,z,t)$ and a uniform temperature rise ΔT, as shown in Fig. 1; here (x,θ,z) denotes the orthogonal coordinate system fixed at the midplane of the nanoshell. The piezoelectric nanoshell is polarized along the thickness direction only.

Based on the Kirchhoff–Love hypothesis, the displacements of an arbitrary point in the shell along the x-, θ- and z-axes, denoted by $u_x(x,\theta,z,t)$, $u_\theta(x,\theta,z,t)$ and $u_z(x,\theta,z,t)$, respectively, are [41,42]

$$u_x(x,\theta,z,t) = U(x,\theta,t) - z\frac{\partial W(x,\theta,t)}{\partial x}, \tag{7}$$

$$u_\theta(x,\theta,z,t) = V(x,\theta,t) - z\frac{\partial W(x,\theta,t)}{\partial \theta}, \tag{8}$$

$$u_z(x,\theta,z,t) = W(x,\theta,t), \tag{9}$$

where $U(x,\theta,t)$, $V(x,\theta,t)$ and $W(x,\theta,t)$ are the displacements in the midplane; and t is the time.

Following Wang [43], the electric potential is assumed to vary as a combination of a cosine and linear variation, which satisfies the Maxwell equation. It can be written as

$$\tilde{\Phi}(x,\theta,z,t) = -\cos(\beta z)\Phi(x,\theta,t) + \frac{2z\phi_0}{h}, \tag{10}$$

where $\beta = \pi/h$; $\Phi(x,\theta,t)$ is the spatial and time variation of the electric potential in the x- and θ- directions; and ϕ_0 is the initial applied external electric voltage.

Using Love's first approximation shell theory, the strain components are computed as

$$\varepsilon_x = \frac{\partial U}{\partial x} - z\frac{\partial^2 W}{\partial x^2}, \tag{11}$$

$$\varepsilon_\theta = \frac{1}{R}\left(\frac{\partial V}{\partial \theta} + W\right) - \frac{z}{R^2}\left(\frac{\partial^2 W}{\partial \theta^2} - \frac{\partial V}{\partial \theta}\right), \tag{12}$$

$$\gamma_{x\theta} = \frac{\partial V}{\partial x} + \frac{1}{R}\frac{\partial U}{\partial \theta} - \frac{z}{R}\left(\frac{2\partial^2 W}{\partial x\partial \theta} - \frac{\partial V}{\partial x}\right). \tag{13}$$

According to the assumed electric potential in Eq. (10), the electric field components can be expressed as [44,45]

$$E_x = -\frac{\partial \tilde{\Phi}}{\partial x} = \cos(\beta z)\frac{\partial \Phi}{\partial x}, \tag{14}$$

$$E_\theta = -\frac{1}{R+z}\frac{\partial \tilde{\Phi}}{\partial \theta} = \frac{\cos(\beta z)}{R+z}\frac{\partial \Phi}{\partial \theta}, \tag{15}$$

$$E_z = -\frac{\partial \tilde{\Phi}}{\partial z} = -\beta\sin(\beta z)\Phi - \frac{2\phi_0}{h}. \tag{16}$$

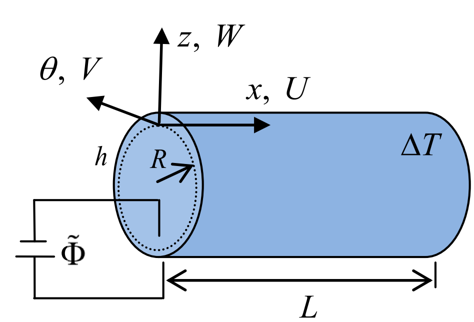

Fig. 1. Schematic configuration of a piezoelectric cylindrical nanoshell.