Composite Structures 116 (2014) 694–702

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Physical interpretation of multiscale asymptotic expansion method

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ARTICLE INFO

Article history:
Available online 14 June 2014

Keywords:
Periodical composite structure
Multiscale
Asymptotic expansion method
Physical interpretation
Influence function

ABSTRACT

The multiscale asymptotic expansion method (MsAEM) up to any expansion order is formulated in a matrix form which is convenient for use as standard finite element method (FEM). Physical interpretations of the influence functions of different order are presented by analyzing the properties of self-balanced quasi load vectors used for solving the influence functions. The physical interpretation of MsAEM will lay the foundation for its applications. Numerical results validate the mathematical formulations and show that the second perturbation is necessary for micro analysis of periodical composite structures.

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1. Introduction

It is well known that iso-strain or iso-stress model [1] and other homogenized approaches [2] can be used to obtain the macro solutions like the lower order frequencies for many composites to the satisfactory accuracy. Compared with the macro analysis, the micro analysis as the strength analysis is very expensive. For balancing accuracy and efficiency, several multiscale methods have been motivated, such as the mathematical homogenization method (MHM) [3,4], the generalized finite element method (GFEM) [5,6], the multiscale finite element (MsFEM) [7,8], the heterogeneous multiscale method (HMM) [9,10] and the multiscale eigenelement method (MEM) [11,12], among of which MHM is representative and has been elaborated in many Refs. [13–21, for examples]. As a representative of MHM, the multiscale asymptotic expansion method (MsAEM, it is also called asymptotic homogenization method) is a powerful technique for the study of heterogeneous media and has been used for solving different kinds of physical problems. But no other paper investigated the physical interpretation of MsAEM except the author's work [11] which discussed the physical implication of the first order perturbation of MsAEM. It is believed that a deep understanding of the physical foundation is of significance for the practical application of MsAEM.

In this context, a compact matrix form of MsAEM is given first, and then a unit cell with multi inclusions of a two-dimensional (2D) periodical composite structure and a periodical composite rod are taken into account to interpret MsAEM in a physical sense. It is noteworthy that, for a unit cell of one-dimensional (1D) periodical composite rod, MsAEM can be interpreted explicitly and analytically. That is why such simple model is involved in the third section.

The outline of present paper is as follows: The matrix form of MsAEM is presented in Section 2, and the physical interpretations of influence functions for different order expansion are investigated in Section 3. Then numerical experiments are conducted in Section 4. Finally, conclusions are drawn in Section 5.

2. Asymptotic homogenization method in a matrix form

Based on the assumptions of microstructure periodicity and uniformity of a unit cell domain, the homogenization theory decomposes the heterogeneous boundary value problem into the unit cell (micro) problem and the global (macro) problem. A 2D periodical composite problem is taken into account below to show the matrix form of MsAEM which is convenient for its use.

The governing equation for 2D composite problem is elliptic for most cases with multiscale or rough coefficients as

$$\begin{aligned} &-\frac{\partial}{\partial x_j}\left(E^{\varepsilon}_{ijmn}(\boldsymbol{x})\frac{1}{2}\left(\frac{\partial u^{\varepsilon}_m}{\partial x_n}+\frac{\partial u^{\varepsilon}_n}{\partial x_m}\right)\right)=f_i(\boldsymbol{x}) \quad \text{in} \quad \Omega\subset R^3 \\ &\boldsymbol{u}^{\varepsilon}(\boldsymbol{x})=\boldsymbol{0} \qquad\qquad\qquad\qquad\qquad\quad \text{on} \quad \partial\Omega \end{aligned} \tag{1}$$

where E^{ε}_{ijmn} is the fourth order elastic tensor, the indices $i,j,m,n=1,2$, the small parameter ε indicates the proportion between the dimensions of a unit cell and the entire domain. Although MHM was formulated and used by many researchers [3–4,13–21], its physical interpretation was rarely touched except a paper [11] of present authors.

The actual displacement u^{ε}_m in asymptotic expansion form is a function of macro and micro scales as

$$u^{\varepsilon}_m(\boldsymbol{x})=u^0_m(\boldsymbol{x})+\varepsilon u^1_m(\boldsymbol{x},\boldsymbol{y})+\varepsilon^2 u^2_m(\boldsymbol{x},\boldsymbol{y})+\cdots \tag{2}$$

where the homogenized displacement u^0_m is offered by the homogenized model, and the perturbed displacement $u^j_m(\boldsymbol{x},\boldsymbol{y})$ of both scales is periodic in $\boldsymbol{y}$; ε is the ratio of the dimensions of a unit cell

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http://dx.doi.org/10.1016/j.compstruct.2014.06.004
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and the entire domain, i.e., ε is given if the sizes of unit cell and entire domain are given.

The perturbed displacements in separation of variables form or uncoupled form and the different order of governing equations for influence functions are listed in Table 1. The influence functions periodical in $\boldsymbol{y}$ depend only on the scale $\boldsymbol{y}$ while the homogenized displacement and its derivatives of different orders only on the scale $\boldsymbol{x}$. In Table 1, the operators $\nabla_{y_j}\bullet$ and ∇_{y_j} in governing equations denote the operation of divergence and gradient in the coordinate frame $\boldsymbol{y}$, respectively. We had many observations from the governing equations of influence functions and expressions of perturbed displacements in uncoupled form in Table 1 [22], one of them is reviewed below for better understanding of present work.

The right side term $\nabla_{y_j}\cdot E^{\varepsilon}_{ijmn}$ in governing equation for the first order influence functions is linear distributed quasi-load which is self-balanced and formed only by the material constants, the load is non-zero along boundary and interface of matrix and inclusion; the right side term for the second order influence functions is self-balanced surface quasi-load, the component $E^{\varepsilon}_{ipkl}-E^{\mathrm{H}}_{ipkl}$ (E^{H} is the homogenized elastic constant) of which depends on material constants only; but the right side terms for solving the third or higher order influence functions have not the components formed only by material constants. Therefore it follows that the first and second order perturbations are necessary, and the neglect of the second order perturbation may cause unacceptable error, that means we have to take the second order perturbation term into account in the use of MsAEM.

The application of MsAEM involves three steps: the first is to solve unit cell problem to obtain influence functions and homogenized elastic constant E^{H}, then to solve the macro or global problem to obtain u^0 and its derivatives, finally to calculate the micro actual displacements and stresses etc.

The unit cell problem is governed by equations in the third column of Table 1. The discretization of these equations gives their matrix form as

$$\boldsymbol{K}^{\varepsilon}\boldsymbol{\chi}_r=\boldsymbol{F}_r \tag{3}$$

where $r=1,2,\ldots$, is the expansion order, the micro stiffness matrix $\boldsymbol{K}^{\varepsilon}$, the influence function vector $\boldsymbol{\chi}_r$ and the self-balanced quasi load matrix $\boldsymbol{F}_r$ have the following forms

$$\boldsymbol{K}^{\varepsilon}=\sum_{e=1}\int_{D^e}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{E}^{\varepsilon}\boldsymbol{B}\mathrm{d}D^e \tag{4}$$

$$\boldsymbol{\chi}_r=\sum_{e=1}\boldsymbol{\chi}^e_r \tag{5}$$

$$\begin{aligned}
\boldsymbol{F}_1&=\sum_{e=1}\int_{D^e}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{E}^{\varepsilon}\mathrm{d}D^e\\
\boldsymbol{F}_2&=\sum_{e=1}\Bigg\{\int_{D^e}\boldsymbol{N}^{\mathrm{T}}\Big[\Big(\boldsymbol{E}^{\mathrm{H}}_1-\boldsymbol{E}^{\varepsilon}_1\Big)\quad\Big(\boldsymbol{E}^{\mathrm{H}}_2-\boldsymbol{E}^{\varepsilon}_2\Big)\Big]\mathrm{d}D^e\\
&\quad+\int_{D^e}\Big[\Big(\boldsymbol{N}^{\mathrm{T}}\boldsymbol{E}^{\varepsilon}_1\boldsymbol{B}-\boldsymbol{B}^{\mathrm{T}}\boldsymbol{E}^{\varepsilon}_3\boldsymbol{N}\Big)\boldsymbol{\chi}^e_1\quad\Big(\boldsymbol{N}^{\mathrm{T}}\boldsymbol{E}^{\varepsilon}_2\boldsymbol{B}-\boldsymbol{B}^{\mathrm{T}}\boldsymbol{E}^{\varepsilon}_4\boldsymbol{N}\Big)\boldsymbol{\chi}^e_1\Big]\mathrm{d}D^e\Bigg\}\\
\boldsymbol{F}_s&=\sum_{e=1}\Bigg\{\int_{D^e}\boldsymbol{N}^{\mathrm{T}}\big[\boldsymbol{E}^{\varepsilon}_5\boldsymbol{N}\boldsymbol{\chi}^e_{s-2}\quad\boldsymbol{E}^{\varepsilon}_6\boldsymbol{N}\boldsymbol{\chi}^e_{s-2}\quad\boldsymbol{E}^{\varepsilon}_7\boldsymbol{N}\boldsymbol{\chi}^e_{s-2}\quad\boldsymbol{E}^{\varepsilon}_8\boldsymbol{N}\boldsymbol{\chi}^e_{s-2}\big]\mathrm{d}D^e\\
&\quad+\int_{D^e}\boldsymbol{N}^{\mathrm{T}}\big[\boldsymbol{E}^{\varepsilon}_1\boldsymbol{B}\boldsymbol{\chi}^e_{s-1}\quad\boldsymbol{E}^{\varepsilon}_2\boldsymbol{B}\boldsymbol{\chi}^e_{s-1}\big]\mathrm{d}D^e-\int_{D^e}\boldsymbol{B}^{\mathrm{T}}\big[\boldsymbol{E}^{\varepsilon}_3\boldsymbol{N}\boldsymbol{\chi}^e_{s-1}\quad\boldsymbol{E}^{\varepsilon}_4\boldsymbol{N}\boldsymbol{\chi}^e_{s-1}\big]\mathrm{d}D^e\Bigg\}
\end{aligned} \tag{6abc}$$

where

$$\boldsymbol{E}^{\mathrm{H}}=\frac{1}{D}\sum_{e=1}\int_{D_e}\boldsymbol{E}^{\varepsilon}(\boldsymbol{I}-\boldsymbol{B}^{\varepsilon}\boldsymbol{\chi}^e_1)\mathrm{d}D_{\mathrm{e}} \tag{7}$$

and $s\geqslant 3$, the integer e is the sub-element number of unit cell model, D^e denotes the domain of sub-element e in unit cell model, the elastic constant matrices used in Eqs. (4) and (6) are given in Appendix A. The dimensions of geometrical matrix $\boldsymbol{B}$, influence matrix $\boldsymbol{\chi}$ and shape function matrix $\boldsymbol{N}$ depend on the element order used in FEM. For a bilinear rectangular sub-element in unit cell model, $\boldsymbol{N}$ is 2 by 8 and has the form as

$$\boldsymbol{N}=\begin{bmatrix}N_1&0&N_2&0&N_3&0&N_4&0\\0&N_1&0&N_2&0&N_3&0&N_4\end{bmatrix} \tag{8}$$

Then $\boldsymbol{B}$ is 3 by 8, $\boldsymbol{\chi}^e_1$ is 8 by 3, $\boldsymbol{\chi}^e_2$ is 8 by 6 and $\boldsymbol{\chi}^e_3$ is 8 by 12. And $\boldsymbol{B}$ has the same form as that in standard FEM, N_i ($i=1,2,3,4$) in Eq. (8) are the node shape functions of a sub-element.

In Table 1, the first order influence function is denoted by χ^{kl}_{1m} (here superscript e is neglected, $r=1$).The superscripts k and l (k, $l=1,2$) have three different combinations as $kl=[11(k=1,\ l=1)$, $22(k=2,\ l=2)$, $12(k=1,\ l=2,$ or $k=2,\ l=1)]$, these three combinations correspond to the three columns of $\boldsymbol{\chi}_1$ in order. Similarly, the six combinations [111,221,121,112,222,122] of klp correspond to the six columns of $\boldsymbol{\chi}_2$ in order, and the twelve combinations [1111,2211,1211,1121,2221,1221,1112,2212,1212, 1122,2222,1222] of $klpq$ to the twelve columns of $\boldsymbol{\chi}_3$ in order.

For macro problem, we have

$$\boldsymbol{K}^{\mathrm{H}}\boldsymbol{u}^0=\boldsymbol{F}_0 \tag{9}$$

$$\begin{aligned}
\boldsymbol{K}^{\mathrm{H}}&=\sum_{g=1}\int_D\boldsymbol{B}^{\mathrm{T}}\boldsymbol{E}^{\mathrm{H}}\boldsymbol{B}\mathrm{d}D\\
\boldsymbol{F}_0&=\sum_{g=1}\int_D\boldsymbol{N}^{\mathrm{T}}\boldsymbol{f}\mathrm{d}D
\end{aligned} \tag{10}$$

where $D\subset\Omega$ represents the domain of a unit cell for a specified periodic composite problem, the integer g is the number of a macro element, generally one takes a unit cell as a macro element.

After solving macro displacements and their derivatives as well as the influence functions, the actual displacements can be given in a matrix form as

$$\boldsymbol{u}^{\varepsilon}=\boldsymbol{u}^0-\varepsilon\boldsymbol{\chi}_1\frac{\partial\boldsymbol{u}^0}{\partial\boldsymbol{x}}-\varepsilon^2\boldsymbol{\chi}_2\frac{\partial^2\boldsymbol{u}^0}{\partial\boldsymbol{x}^2}-\varepsilon^3\boldsymbol{\chi}_3\frac{\partial^3\boldsymbol{u}^0}{\partial\boldsymbol{x}^3}+\cdots \tag{11}$$

where the forms of $\partial\boldsymbol{u}^0/\partial\boldsymbol{x}$, $\partial^2\boldsymbol{u}^0/\partial\boldsymbol{x}^2$ and $\partial^3\boldsymbol{u}^0/\partial\boldsymbol{x}^3$ can be readily determined by using the forms of $\boldsymbol{\chi}_1$, $\boldsymbol{\chi}_2$ and $\boldsymbol{\chi}_3$ in conjunction with the combinations of kl, klp and $klpq$ as given above.

It can be seen from above formulae that all equations in uncoupled MsAEM can be solved by means of the finite element method, and a salient feature of MsAEM is that the fine scale solution is completely described on the coarse scale, see Eq. (11), that means the accuracy of the actual solutions depends on the macro displacements $\boldsymbol{u}^0$ and their derivatives $\partial\boldsymbol{u}^0/\partial\boldsymbol{x}$, $\partial^2\boldsymbol{u}^0/\partial\boldsymbol{x}^2$ and $\partial^3\boldsymbol{u}^0/\partial\boldsymbol{x}^3$ once the influence functions are solved over a representative volume cell.

Table 1
Perturbed displacements and governing equations for influence functions.

Order	Perturbed displacements	Governing equations for influence functions
1	$u^1_m(\boldsymbol{x},\boldsymbol{y})=-\chi^{kl}_{1m}(\boldsymbol{y})\frac{\partial u^0_k(\boldsymbol{x})}{\partial x_l}$	$\nabla_{y_j}\cdot\left(E^{\varepsilon}_{ijmn}\nabla_{y_n}\chi^{kl}_{1m}\right)=\nabla_{y_j}\cdot E^{\varepsilon}_{ijmn}$
2	$u^2_m(\boldsymbol{x},\boldsymbol{y})=-\chi^{klp}_{2m}(\boldsymbol{y})\frac{\partial^2 u^0_k(\boldsymbol{x})}{\partial x_l\partial x_p}$	$\nabla_{y_j}\cdot E^{\varepsilon}_{ijmn}\nabla_{y_n}\chi^{klp}_{2m}=-\nabla_{y_j}\cdot E^{\varepsilon}_{ijmp}\chi^{kl}_{1m}-E^{\varepsilon}_{ipmn}\nabla_{y_n}\chi^{kl}_{1m}+E^{\varepsilon}_{ipkl}-E^{\mathrm{H}}_{ipkl}$
3	$u^3_m(\boldsymbol{x},\boldsymbol{y})=-\chi^{klpq}_{3m}(\boldsymbol{y})\frac{\partial^3 u^0_k(\boldsymbol{x})}{\partial x_l\partial x_p\partial x_q}$	$\nabla_{y_j}\cdot E^{\varepsilon}_{ijmn}\nabla_{y_n}\chi^{klpq}_{3m}=-E^{\varepsilon}_{iqmn}\nabla_{y_n}\chi^{klp}_{2m}-\nabla_{y_j}\cdot E^{\varepsilon}_{ijmq}\chi^{klp}_{2m}-E^{\varepsilon}_{iqmp}\chi^{kl}_{1m}$

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