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Nonlinear vibration analysis of piezoelectric nanoelectromechanical resonators based on nonlocal elasticity theory

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ABSTRACT

In this paper, a nonlinear continuum model is developed for the large amplitude vibration of nanoelectromechanical resonators using piezoelectric nanofilms (PNFs) under external electric voltage. Hamilton's principle in conjunction with von Karman's theory is employed to derive the differential equations of motion. Size effects are incorporated into both the governing equations and in-plane boundary conditions using nonlocal continuum mechanics. Explicit expressions are presented for the nonlinear natural frequencies and critical electric voltages of PNFs. In comparison to the available experimental data and molecular dynamics simulation results, the present nonlocal model with reasonable small scale parameters results in more accurate estimation of natural frequencies than the classical theory of plates. It is anticipated that the results of the present work would be helpful in experimental characterization of the mechanical properties of piezoelectric nanoresonators.

1. Introduction

Recently, nanoresonators have attracted a great deal of attention from research communities because of their promising applications in nanoelectromechanical systems (NEMS) [1–5]. They can be applied in the detection of amyloid growth [6], quantum ground state [7] and spin manipulation [8]. More details about the nanoresonator-based detections and their nanomechanics principles can be found in the review paper of Eom et al. [3]. The nanoelectromechanical resonators can also be used for the measurement of mass, density and volume during the cell cycle of yeast [9]. Another potential application of these nanodevices is the attonewton-scale force resolution [10]. In the operation of NEMS devices, the actuation and detection of nanometer motion at high frequencies is one of the most important challenges. Ekinci [1] reviewed the electromechanical transducers at nanoscales and available techniques to actuate and detect NEMS motion. Further, it has been shown that very high frequency NEMS resonators provide unprecedented sensitivity in measuring molecular weight [11]. Recently, it has been reported that nanoresonators such as suspended microchannel resonators (SMRs) and optical microring resonators (MRRs) have potential application in the next-generation mechanical biosensors [4]. The ability of a resonator to detect a physical quantity depends on its resonant frequency. For example, the ability of the resonator to sense small molecules increases with increasing the resonant frequency [12]. The identification of small molecules would be useful in the early and efficient diagnosis of diseases such as cancer [13].

Piezoelectric nanostructures such as GaN nanowires [14] and aluminum nitride (AlN) thin films [15] can be used as building blocks for nanoresonators. Sinha et al. [16] showed that the superior material properties of ultrathin AlN piezoelectric films make them ideal candidates for the fabrication of nanoelectromechanical switches. In addition, Briscoe et al. [17] presented measurement techniques for electromechanical energy harvesting at small scale using piezoelectric nanostructures. For more information about the possible applications of piezoelectric nanostructures in nanotechnology, we refer readers to the review paper of Fang et al. [18]. Due to these promising applications, the increasing level of knowledge of the mechanical characteristics of piezoelectric nanostructures is vital for the proper design and fabrication of smart nanodevices.

During the past decade, different higher-order theories have been used to study the bending, vibration and buckling of micro/nano-structural elements such as carbon nanotubes [19], nanorings [20], microtubules [21] and graphene sheets [22]. Demir et al. [19] studied the vibration characteristics of carbon nanotubes based on shear deformable beam theory and discrete singular

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convolution technique. Civalek and Akgoz [21] investigated the free vibration analysis of microtubules using nonlocal Euler–Bernoulli beam theory. A review of literature shows that, compared to the carbon nanotubes and graphene sheets, few research works have been carried out on the theoretical investigation of piezoelectric nanostructures, especially on the nonlinear vibration properties. The influences of residual surface stress, surface elasticity and surface piezoelectricity on the vibration and buckling of piezoelectric nanobeams were studied based on the Euler–Bernoulli beam theory [23]. Yan and Jiang [24] developed a theoretical model incorporating the effects of surface energy to predict the electromechanical response of a curved piezoelectric nanobeam. They also examined the electroelastic response of piezoelectric nanofilms under electromechanical loads with the consideration of surface effects [25]. Based on the nonlocal continuum mechanics, modified linear and nonlinear beam models were developed to study the small scale effects on the small [26] and large amplitude [27] vibrations of piezoelectric nanobeams, respectively. Furthermore, Liu et al. [28] determined the linear natural frequencies of piezoelectric nanoplates under thermo-electro-mechanical loading with the use of the nonlocal elasticity theory. More recently, the influence of voltage distribution on the nonlocal linear free vibration of coupled piezoelectric-nanoplate-systems embedded in Pasternak elastic medium has been studied [29].

Generally, the amplitude of vibration and static deflection of a nanostructure may be larger than the order of its thickness. In such cases, the linear continuum models are uncertain in order to accurately obtain the natural frequencies and critical buckling loads. Recently, Farajpour et al. [30] have examined the postbuckling of multi-layered graphene sheets (MLGS) under non-uniform biaxial compression including nonlinear van der Waals interactions. Nazemnezhad and Hosseini-Hashemi [31] developed a nonlocal nonlinear beam model for the free vibration of functionally graded nanobeams. Furthermore, Golmakani and Rezatalab [32] investigated the nonlinear bending of orthotropic nanoplates embedded in an elastic medium using nonlocal continuum mechanics. However, to the authors' best knowledge, up to now, the nonlinear vibration of NEMS resonators using piezoelectric nanofilms (PNFs) under external electric voltage has not been studied in the literature. This motivates us to investigate this problem here. The small scale effects are incorporated into the governing equations and in-plane boundary conditions by applying the nonlocal continuum mechanics to the classical plate theory. Three coupled nonlinear differential equations are derived for transverse vibrations using von Karman nonlinear model and Hamilton's principle. Analytical solutions are obtained for the nonlinear natural frequencies and critical electric voltages of PNFs with simply supported boundary conditions. The present results are compared with the available experimental data and atomistic simulation results and an excellent agreement is found. It is shown that the natural frequencies of piezoelectric nanoresonators can be tuned by adjusting the value of external electric voltage.

2. Nonlocal nonlinear plate model for PNFs

In this section, we develop a nonlocal nonlinear plate model for the large amplitude vibration of piezoelectric NEMS resonators. Fig. 1 shows a nanoelectromechanical resonator consists of a rectangular PNF with uniform thickness subjected to an external electric voltage. A Cartesian coordinate frame is employed to label the material points of the system. The x, y and z axes of the coordinate frame are assumed along the length (ℓ_x), width (ℓ_y) and thickness (h) of the nanofilm, respectively.

The size dependence of mechanical behavior has been observed in nanosized structural elements such as carbon nanotubes [33,34], nanorods [35,36], microtubules [37], nanoparticles [38] and

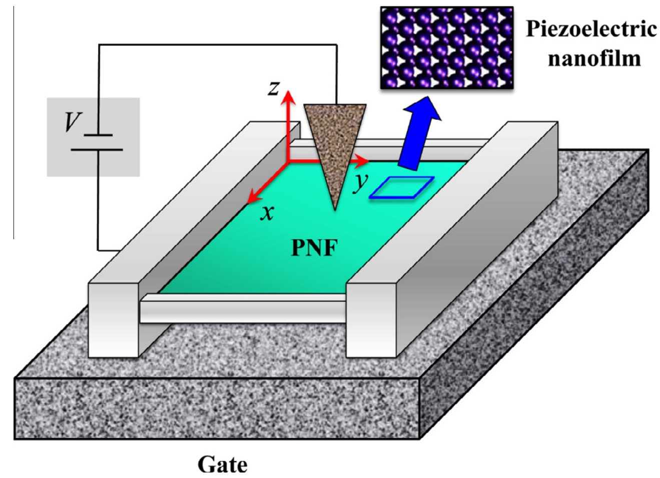

Fig. 1. Schematic representation of a piezoelectric nanoelectromechanical resonator.

nanoplates [39–41]. Nonlocal continuum models take into account the long-range interatomic interactions and thus yield size-dependent results [42]. This theory is based on a simple physical concept that the components of stress tensor at a given point are a function not only of strain tensor at that point but also are a function of strain tensors at all other points in the domain. The results of molecular dynamics simulations and experimental data on phonon dispersion demonstrated that the nonlocal continuum models are reasonable to obtain the mechanical characteristics of nanostructures [30,39,42]. Based on the nonlocal continuum mechanics, the stress–strain relations for Hookean piezoelectric solids neglecting the body force can be expressed as

$$\sigma_{ij} = \int\int_V\int H(|x' - x|, \chi)[c_{ijkl}\varepsilon_{kl}(x') - e_{kij}E_k(x')]dx' \quad (1)$$

$$D_i = \int\int_V\int H(|x' - x|, \chi)[e_{ikl}\varepsilon_{kl}(x') + \kappa_{kij}E_k(x')]dx' \quad (2)$$

$$\sigma_{ij,j} = \rho\ddot{u}_i, \quad D_{i,j} = 0, \quad E_i = -\Phi_{,i} \quad (3\text{a-c})$$

where σ_{ij} and D_i are the nonlocal stress and electric displacement, respectively; ε_{ij}, E_i and U_i are the components of strain tensor, electric field vector and displacement vector, respectively; Φ represents the electric potential; Also, the terms c_{ijkl}, e_{kij}, κ_{kij} and ρ are respectively the fourth order elasticity tensor, piezoelectric constants, dielectric constants and mass density of the PNF. The term $H(|x' - x|, \chi)$ is the nonlocal modulus in which $|x - x'|$ is the distance between points x and x' and $\chi = e_0\ell_i/\ell_e$ is the small scale coefficient. ℓ_i and ℓ_e are the internal and external characteristic lengths, respectively. Choice of the value of parameter e_0 is crucial for the validity of nonlocal models. This parameter can be obtained by matching the dispersion curves of plane waves with those of atomic lattice dynamics [42]. In other words, results can be justified by an approximation of the atomic dispersion relations. Due to the mathematical difficulties associated with using Eqs. (1) and (2) in the formulation of nanosized structural elements, the following stress–strain relations are often used [26–37]

$$\sigma_{ij} - (e_0\ell_i)^2\nabla^2\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k,$$
$$D_i - (e_0\ell_i)^2\nabla^2 D_i = e_{ikl}\varepsilon_{kl} + \kappa_{kij}E_k \quad (4\text{a, b})$$

where $e_0\ell_i$ is the nonlocal parameter which contains the small scale effects. ∇^2 is the Laplacian operator and is given by $\nabla^2(*) = \partial^2(*)/\partial x^2 + \partial^2(*)/\partial y^2$. Using the above relation, the nonlocal