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Static analysis of higher order sandwich beams by weak form quadrature element method

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A B S T R A C T

Based on the extended high order sandwich panel theory (EHSAPT) and differential quadrature rule, an *N*-node novel weak form quadrature sandwich beam element is established. Gauss Lobatto Legendre (GLL) points are utilized as element nodes and GLL quadrature is used to obtain the element stiffness matrix and work equivalent load. Detailed formulations are given. Convergence study is performed. Numerical results are presented for sandwich beams with different boundary conditions subjected to distributed loadings. Different materials for the face sheets and core of the beam structure are considered. It is shown that the proposed beam element can yield very accurate displacements and stresses as compared to theoretical solutions.

1. Introduction

Sandwich structures are of high stiffness and strength to weight ratio, as well as its energy absorption capability [1,2], which accounts for its prevalence in fields of aerospace, naval and civil constructions. Especially its low weight penalty is of vital importance to the weight sensitive fields like aeronautics and astronautics [3]. Thus they have been received great attention theoretically and numerically.

The earliest classical (CL) theory model was established based on Euler–Bernoulli theory, the transverse stiffness of the core was considered as infinitely rigid. First order shear deformation (FOSD) theory was established based on Timoshenko beam theory; the shear deformation of the core is taken into considerations. However, the accuracy of FOSD theory is not persuasive when comparing with the elasticity solution. The high-order sandwich panel theory (HSAPT) takes into account the in-plane rigidity of the core and satisfies the elasticity solution. With the progress of experimental study [4] and theoretical development [5,6], the influence of the soft core has been gradually taken into account. To eliminate the error caused by the very soft core configuration, a new theory, called extended high-order sandwich panel theory (EHSAPT), has been proposed recently [7]. EHSAPT has been verified in static as well as in dynamic analysis, which showed its capability in predicting not only accurate displacements and stresses in static analysis, but also blast responses and global buckling problems. Due to the complicated mathematical structure of the theory, however, it is not an easy task to obtain closed form solutions for beams with different boundary conditions subjected to general loadings. Therefore, it is of great necessity to develop an efficient and accurate numerical computational method based on the EHSAPT.

Besides the widely used finite element method (FEM), a variety of new and efficient methods have been developed in recent years [8–41]. To name a few, strong form differential quadrature method (DQM or GDQM) [8–14] and harmonic differential quadrature method (HDQM) [15,16], discrete singular convolution (DSC) [17], time domain spectral finite element method (SFEM) [18], the moving least square-Ritz method (MLS-Ritz) [19], strong form differential quadrature element method (DQEM) [20–32], and weak form quadrature element method (WQEM) [33–41]. It is seen that both strong form and weak form differential quadrature methods can handle complex geometries and overcome the shortcomings existing in the ordinary DQM, thus greatly extended the application range of the DQM in the area of structural mechanics.

Among the two approaches, the strong form differential quadrature method has advantages of faster accuracy and easier writing of the governing equations. Its main drawback is difficult to implement the multiple boundary conditions. On the contrary, the weak form differential quadrature method is much easier to implement the multiple boundary conditions. Similar to the FEM and SFEM, the weak form quadrature element method is also based on the principle of minimum potential energy, but uses the available explicit formulations to compute the weighting coefficients of derivatives at integration points. The stiffness matrix of the WQEM

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is symmetric. Compared to the DQEM, however, WQEM may take more computational time for the implementation of the governing equations. The method needs two calculations, one for the derivatives and the other for the integration.

It is seen that to formulate a weak form quadrature element, the derivatives at integration points are computed explicitly by using the differential quadrature rule, thus their explicit expressions of shape functions are not needed. This will greatly reduce the effort of programming as well as the complexity in presentation. With the newly proposed method [41], formulations of the weak form quadrature element do not have any difficulty even the integration points are not exactly the same as the nodal points. Since the formulation of the weak form quadrature element is more simple and flexible as compared to the FEM, therefore, the WQEM is to be used in the present investigations.

The objective of this article is to establish a novel N-node weak form quadrature sandwich beam element based on the extended high order sandwich panel theory. Gauss Lobatto Legendre (GLL) points are adopted as the elemental nodes and GLL quadrature is used to obtain the element stiffness matrix and work equivalent load. Several examples are investigated. Results are compared with available theoretical solution for verifications. Some new results are also provided for reference. Conclusions are drawn based on the results reported herein.

2. High-order sandwich panel theory

For completeness consideration, the extended high-order sandwich panel theory (EHSAPT) [7] is briefly introduced. The sandwich beam, schematically shown in Fig. 1, has a length of a, and thicknesses of top and bottom faces, and the core are f_t, f_b and $2c$, respectively.

The top and bottom faces are modeled as Euler–Bernoulli beams. The displacements are given by

$$w^t(x,z) = w_0^t(x) \tag{1}$$

$$u^t(x,t) = u_0^t(x) - \left(z - c - \frac{f_t}{2}\right) w_{0x}^t(x) \tag{2}$$

$$w^b(x,z) = w_0^b(x) \tag{3}$$

$$u^b(x,t) = u_0^b(x) - \left(z + c + \frac{f_b}{2}\right) w_{0x}^b(x) \tag{4}$$

where superscripts t and b refer to the top and bottom faces, $w_0^t(x)$ and $w_0^b(x)$ are transverse displacement of the middle plane of the top and bottom faces, $u_0^t(x)$ and $u_0^b(x)$ are longitudinal displacement of the middle plane of the top and bottom faces, and the subscript x denotes the first order derivative with respect to x.

The core is modeled as a two-dimensional plate. The displacements of the core are assumed as

$$u^c(x,z) = u_0^c(x) + \phi_0^c(x)z + u_2^c(x)z^2 + u_3^c(x)z^3 \tag{5}$$

$$w^c(x,z) = w_0^c(x) + w_1^c(x)z + w_2^c(x)z^2 \tag{6}$$

where superscript c refers to the core and subscript 0 refers to its middle surface, x–z is the coordinated set on the middle surface of the sandwich panel, as shown in Fig. 1, $u_0^c(x)$ and $w_0^c(x)$ are longitudinal and transverse displacement, and $\phi_0^c(x)$ is the slope at the centroid of the core, $u_2^c(x)$, $u_3^c(x)$, $w_1^c(x)$ and $w_2^c(x)$ are four undetermined coefficients, respectively.

The four undetermined coefficients in Eqs. (5) and (6) can be eliminated by applying compatibility requirements in the transverse direction on the top and bottom face-core interfaces. After doing so, Eqs. (5) and (6) are given by

$$\begin{aligned} u^c(x,z) = {} & z\left(1 - \frac{z^2}{c^2}\right)\phi_0^c(x) + \frac{z^2}{2c^2}\left(1 - \frac{z}{c}\right)u_0^b(x) + \left(1 - \frac{z^2}{c^2}\right)u_0^c(x) \\ & + \frac{z^2}{2c^2}\left(1 + \frac{z}{c}\right)u_0^t(x) + \frac{f_b z^2}{4c^2}\left(-1 + \frac{z}{c}\right)w_{0x}^b(x) \\ & + \frac{f_t z^2}{4c^2}\left(1 + \frac{z}{c}\right)w_{0x}^t(x) \end{aligned} \tag{7}$$

$$\begin{aligned} w^c(x,z) = {} & \left(-\frac{z}{2c} + \frac{z^2}{2c^2}\right)w_0^b(x) + \left(1 - \frac{z^2}{c^2}\right)w_0^c(x) \\ & + \left(\frac{z}{2c} + \frac{z^2}{2c^2}\right)w_0^t(x) \end{aligned} \tag{8}$$

The strain energy for the top and bottom face sheets of the sandwich beam element is

$$\begin{aligned} U^{t,b} & = \frac{1}{2}\int_0^L \left[\frac{E^{t,b} f_{t,b}^3}{12}\left(\frac{\partial^2 w^{t,b}}{\partial x^2}\right)^2 + E^{t,b} f_{t,b}\left(\frac{\partial u^{t,b}}{\partial x}\right)^2\right] dx \\ & = \frac{1}{2}\int_{-1}^1 \left[\frac{E^{t,b} f_{t,b}^3}{12}\frac{8}{L^3}\left(\frac{\partial^2 w^{t,b}}{\partial \xi^2}\right)^2 + \frac{2E^{t,b} f_{t,b}}{L}\left(\frac{\partial u^{t,b}}{\partial \xi}\right)^2\right] d\xi \end{aligned} \tag{9}$$

where t and b denote the top and bottom face sheets, $E^{t,b}$ is the elasticity modulus, and L is the element Length, and $\xi = 2x/L - 1$.

The strain energy for the core of the sandwich beam element is

$$U^c = \frac{1}{2}\int_0^L \int_{-c}^c \{\varepsilon(x,z)\}^T [D] \{\varepsilon(x,z)\} dz\, dx \tag{10}$$

where $\{\varepsilon(x,z)\}$ contains three strains, namely, $\varepsilon_x(x,z)$, $\varepsilon_z(x,z)$, $\gamma_{xz}(x,z)$. The detailed expression can be found in [7]. Since the core material is orthotropic, thus $[D]$ is given by

$$[D] = \begin{bmatrix} -\frac{E_3^c E_1^c}{-E_3^c + \nu_{13}^2 E_1^c} & -\frac{\nu_{13} E_3^c E_1^c}{-E_3^c + \nu_{13}^2 E_1^c} & 0 \\ -\frac{\nu_{13} E_3^c E_1^c}{-E_3^c + \nu_{13}^2 E_1^c} & -\frac{E_3^{c2}}{-E_3^c + \nu_{13}^2 E_1^c} & 0 \\ 0 & 0 & G_{31}^c \end{bmatrix} \tag{11}$$

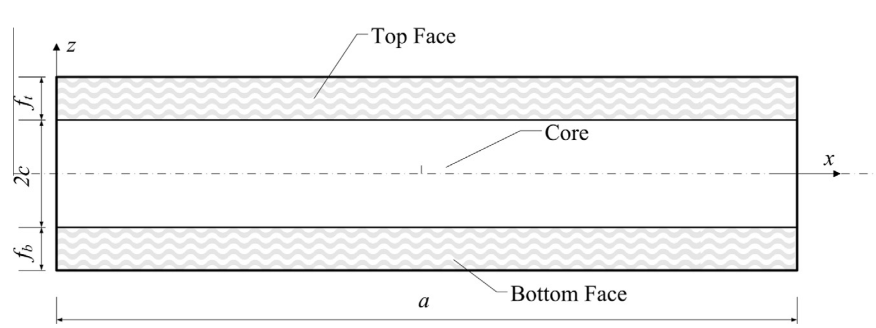

Fig. 1. Scketch of a sandwich beam.