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Dynamic characteristics of laminated thin cylindrical shells: Asymptotic analysis accounting for edge effect

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ABSTRACT

The dynamic characteristics of composite thin cylindrical shells are examined through a systematic order-of-magnitude analysis. The analysis is used to eliminate terms of secondary importance, while retaining the dominant terms in the dispersion relation and boundary conditions. This results in analytical expressions that can describe the vibration of composite cylindrical shells with high accuracy for a wide range of frequencies. Furthermore, the asymptotic analysis is carried out in such a way that the dynamic edge effect is accounted for when determining the vibration mode shapes and the associated internal stresses. Numerical examples are also presented. It is shown that the proposed methodology gives closed-form and analytical results that are in close agreement with numerical solutions of the equations of motion.

1. Introduction

Thin composite shells have been extensively used as structural components in aerospace and automobile industries, nuclear reactors and many other applications. This is due to their favorable material properties such as high strength and stiffness-to-weight ratio. While emerging computational tools for the analysis of composite shells are being constantly developed [27,26], researchers are finding that the exploration of classical shell theories can continue to provide novel insights into these structures [23,36,6]. Love [24] was the first to thoroughly study the vibration of shells with considerations of the effects of both bending and extension. Other researchers have developed different theories for vibration of shells based on Love's postulates. These theories share Kirchhoff's assumption which states that normals to the undeformed midsurface remain straight and normal to the deformed midsurface. What distinguishes these theories are the simplifying assumptions adopted and the point in the development process where these assumptions are used. These result in different shell theories such as Flügge, Love–Timoshenko, Sanders, and Donnell–Mushtari theories, each with its own set of equations of motion. A comprehensive review of different theories of shell vibration is summarized in the classical treatise by Leissa [22].

Among thin shells, cylindrical shells are among the most commonly used in practice. The equations of motion for the free vibration of cylindrical shells, however, do not have closed-form solutions for most boundary conditions and levels of heterogeneity (e.g., orthotropic). One of the only cases where analytical solutions do exist is for freely supported boundary conditions [2,7,4], where the mode shapes can be written as Fourier series in both longitudinal and circumferential directions. As alternatives to closed-form analytical solutions, several semi-analytical and numerical approaches have been proposed to solve for the dynamic characteristics of composite cylindrical shells. One approach is to substitute the exact forms of the vibration mode shapes (exponential in the longitudinal direction and Fourier type in the circumferential direction) in the equations of motion and boundary conditions. This results in a polynomial dispersion relation coupled with a frequency determinant that have to be solved numerically for the natural frequencies and the associated wavenumbers [9,37,7]. This approach was first proposed by Forsberg [9] who solved the Flügge shell equations for isotropic cylindrical shells for several boundary conditions. While these numerical procedures lead to exact solutions for the natural frequencies and mode shapes (up to the accuracy of the numerical solution for the zeros of the coupled dispersion relation determinant), they do not provide an in-depth understanding of the vibration behavior of

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cylindrical shells, especially for non-isotropic shells which have more complicated equations of motion and boundary conditions. Furthermore, solving the characteristic equations may prove to be laborious in some cases [5]. Another approach is to use energy approaches such as the Rayleigh–Ritz method [34,33,21,13] where the vibration mode shapes in the longitudinal direction are written in terms of predefined functions (e.g., beam vibration mode shapes) with undetermined coefficients which are calculated by invoking a variational principle. While this approach is attractive in its simplicity, in many cases, a large number of such predefined functions are needed to accurately account for the edge effect, characterized by the rapid variation of stresses and strains that often occurs near the ends of the cylinder. Moreover, as pointed out by Elishakoff and Wiener [8], while such numerical strategies are computationally efficient in the low frequency range, they become less efficient at high frequencies. The energy approach for the calculation of natural frequencies and mode shapes is sometimes facilitated by adopting additional assumptions, such as zero hoop and shear strains in strain and kinetic energy expressions [31,32] that lead to simpler frequency equations, e.g., quadratic and linear equations instead of sextic and cubic equations.

There has also been success in using simplifying assumptions other than those on the kinematics of deformation to arrive at analytical expressions for the natural frequencies of cylindrical shells. Yu [41], for example, assumed small longitudinal and large circumferential wavenumbers (valid for long cylindrical shells) and used Donnell shell theory to obtain a simple characteristic equation for the natural frequencies for three different boundary conditions. Yu's equations are similar to the ones associated with lateral vibration of an Euler–Bernoulli beam. These equations are accurate for simply supported boundary conditions, but are less accurate for free edges where complex moment and shear conditions that characterize the edge effect need to be satisfied [35]. Later, Soedel [35] used Galerkin's method with general beam mode shapes as the shape functions for isotropic cylinders with various boundary conditions. The use of Galerkin's method eliminates the need for Yu's approximation and results in a simplified expression for natural frequencies of cylindrical shells with accuracy that increases as circumferential wavenumbers grow relative to the longitudinal ones. The results reported in Soedel's work are identical to the expressions proposed by Weingarten [38] but are derived using a different approach.

Another popular approach to calculate the dynamic characteristics of thin cylindrical shells is through the use of asymptotic methods. This approach not only provides accurate solutions in many practical cases, but also can be used to determine the order-of-magnitude relations among the various dimensionless quantities entering the vibration problem. Following the early work of Goldenveizer [11], and Ross [29,30], Nau and Simmonds [25] used the method of composite expansions (MCE) to calculate the natural frequencies in clamped–clamped cylindrical shells. MCE was also used to reduce the order of shell vibration problems (in the presence of non-vanishing pre-stresses) and transform them into equivalent membrane problems [14]. The composite expansions used were shown to be uniformly valid over the length of the shell and accounted for the effects of bending near the edges. Koga [15] used asymptotic expansions of longitudinal wavenumbers in the equations of motion and boundary conditions and obtained general characteristic equations for isotropic thin cylindrical shells for a variety of boundary conditions under the assumption that the vibrations are nearly inextensional. Koga identified two groups of longitudinal wavenumbers: the first group had modal properties that varied gradually over the entire length while the second group was formulated with edge zone solutions that decay rapidly as the distance from the edge increases to approximate the edge effect. Williams [39] applied the method of matched asymptotic expansion (MMAE) to the membrane equations of motion for thin cylindrical shells and obtained the natural frequencies for fixed–fixed and fixed–free end conditions which were shown to be accurate for small circumferential wavenumbers. Unlike the work of Koga, Williams expanded the displacement fields in terms of a boundary layer coordinate in the near-edge region and matched the near-edge solution to the solution away from the edge. Later, Wong and Bush [40] used the method of matched asymptotes and extended the analysis in Killian et al. [14] to obtain analytical results for long cylindrical shells for the axisymmetric mode shapes with frequencies below the ring frequency.

The goal of this paper is to add to the above literature with the derivation of a new set of analytical expressions that describe the free vibration of symmetric and orthotropic composite laminated cylinders with high accuracy for a wide range of frequencies. The challenge here, as noted by previous researchers, is that the equations of motion and boundary conditions, which are complex even for the case of isotropic cylindrical shells, become more involved for laminated composite cylinders. Our approach is to use asymptotic methods to classify the importance of the dominant terms, through a systematic order-of-magnitude analysis.

After briefly reviewing the Love–Timoshenko equations in Section 2, we proceed with an order-of-magnitude analysis of the eighth-order dispersion relation in Section 3.1. It is shown how this relation can be reduced to decoupled bi-quadratic equations, providing relatively simple relationships between the natural frequency and wavenumber for both small and large wavenumbers. Asymptotic analysis is then used in Section 3.2 to derive relatively transparent expressions for the internal forces and moments. In Section 4, these expressions are used to satisfy the boundary conditions leading to simplified analytical expressions for the natural frequencies and mode shapes. Throughout Sections 3 and 4, it is shown how the analytical results for the dispersion relation, boundary forces and free-vibration modal properties compare favorably with numerical solutions based on the original Love–Timoshenko equations, particularly in capturing the dynamic edge effect. Furthermore, it is shown how these analytical results provide insights into the relationships between the dynamic behavior and the parameters that define the composite cylinder. It is noted that these results are useful beyond the theoretical characterization of vibrations of thin cylindrical shells. For instance, they can be used in system identification where guidance is needed to estimate the parameters such as the elastic properties from vibration measurements.

Our approach in this paper is inspired by the work of Bolotin [3], who developed asymptotic solutions with correction terms at the boundaries, and by the extension of this work by Elishakoff and Wiener [8]. Huang and Dong [12] also argued that the edge effect resembles the rapid decay of elastostatic self-equilibrated stress distributions and can be viewed as the dynamic counterpart of Saint Venant's principle. They, however, used a semi-analytical procedure where the displacement field in the radial direction was discretized via quadratic interpolating functions. Here, we specifically draw upon the work of Koga [15] which, although approximate in nature, introduced the concept of partitioning the waveforms into two sets, those that vary gradually over the entire length and those that represent the edge-zone solutions which decay rapidly as the distance from the end increases.

The approach adopted herein is also inspired by recent papers by Ladevéze [18,20] which are based on a significantly different application of Saint Venant's principle: the solution can be partitioned into two parts, one in which the wavelength is large and one in which the wavelength is small and localized in a region (see also [17,19]. A recent work [1] based on the concepts put forward by Ladevéze has focused on capturing edge effects in thermoelastically deforming composite pipes.