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Shear deformation beam models for functionally graded microbeams with new shear correction factors

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ABSTRACT

A shear deformation beam model and new shear correction factors are presented for nonhomogeneous microbeams. The governing equations and corresponding boundary conditions in bending and buckling are obtained by implementing minimum total potential energy principle. Bending and buckling problems of a simply supported functionally graded microbeam are analytically solved by Navier solution procedure. Several comparative results are given for different material property gradient index, thickness-to-material length scale parameter ratio (or vice versa), slenderness ratio and shear correction factors. It is observed that size effect and shear deformation are more significant for lower values of thickness-to-material length scale parameter and slenderness ratios, respectively.

1. Introduction

Microbeams that the characteristic dimensions are on the order of microns and sub-microns, are the key structures of many micro-electro mechanical systems (MEMS) such as micro-resonators [1], Atomic Force Microscopes [2], micro-actuators [3], and micro-switches [4]. Recently, the existence of size effects on deformation behavior associated with microstructure has been experimentally observed with some experiments such as micro-torsion [5] and micro-bending tests [6]. These microstructural effects cannot be taken into account by conventional continuum theories without any additional material length scale parameters. Therefore, several non-classical continuum theories have been proposed to predict the mechanical behaviors of small-scale structures, such as couple stress theory [7–9], micropolar theory [10], nonlocal elasticity theory [11,12] and strain gradient theories [13–16].

The modified strain gradient theory (MSGT) [17] is one of the above-mentioned non-classical theories in which strain energy density contains second-order deformation gradients (dilatation gradient vector, deviatoric stretch and symmetric rotation gradient tensors) in addition to first-order deformation gradient (symmetric strain tensor). For linear elastic isotropic materials, the formulations and governing equations include three additional material length scale parameters related to higher-order deformation gradients besides two classical ones. This popular theory has been employed to investigate mechanical behaviors of size-dependent one-dimensional homogeneous microstructures, such as bars [18–21] and beams [22–27]. More recently, finite element formulations based on this theory have been derived for Bernoulli–Euler and Timoshenko microbeam models [28,29].

Also, in the absence of dilatation gradient vector and deviatoric stretch gradient tensor in formulations and governing equations of MSGT, this theory will be transformed to another non-classical theory called as modified couple stress theory (MCST) [30]. This theory has been used to analyze static and dynamic responses of microbeams [31–39].

Functionally graded materials (FGMs) can be described as a new improved kind of composite materials in which material properties change from one surface to another continuously and smoothly. In conventional laminated composites, high stress concentrations may consist at the interface of two-layer due to sudden change in material properties. On the contrary, undesirable stress concentrations can be prevented in FGMs and they have superior thermo-mechanical properties. Because of FGMs are more advantages than homogeneous and laminated composites, they have a wide range of applications in many engineering fields such as aerospace, biomedicine, nuclear, electronics, optics, and mechanical. Many studies have been performed to investigate static and dynamic responses of beams, plates and shells made of functionally graded materials with various solution methods [40–47]. Recently, use of microstructures made of FGM has increased in many applications as an essential part of MEMS [48], micro-actuators [49] and shape memory alloys [50]. Several studies have been performed to determine mechanical characteristics of nonhomogeneous microbars and microbeams. For instance, Akgöz and Civalek [51] presented

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a study on longitudinal vibration analysis of axially functionally graded strain gradient microbars. Sadeghi et al. [52] investigated the effect of material length scale parameters on analysis of functionally graded micro-cylinders based on MSGT. Kahrobaiyan et al. [53], Akgöz and Civalek [54], Zhang et al. [55], Ansari et al. [56] and Tajalli et al. [57] developed strain gradient Bernoulli–Euler and Timoshenko beam models for FGM microbeams, respectively. More recently, a microstructure-dependent Bernoulli–Euler beam model is developed for bilayered microbeams on the basis of the strain gradient elasticity theory [58].

As a result of literature survey, it can be seen that various beam theories have been proposed to investigate static and dynamic behaviors of beam-type structures. The well-known of these are Euler–Bernoulli (EBT) and Timoshenko (TBT) beam theories. According to assumptions in EBT, effects of shear deformation are ignored. The use of this theory can be suitable for slender beams with a large aspect ratio. However, effects of shear deformation become more prominent for moderately thick beams. TBT [59] known as first-order shear deformation beam theory is an earlier shear deformation beam theory. TBT assumes that transverse shear stress and strain are uniform throughout the height of the beam. Because of the absence of transverse shear stress and strain at the top and bottom surfaces of the beam, a shear correction factor is required in formulations. Subsequently, several higher-order shear deformation beam theories (HBTs) including parabolic (third-order) beam theory (PBT) [60,61], trigonometric (sinusoidal) beam theory (SBT) [62], hyperbolic beam theory [63], exponential beam theory [64] and a general exponential beam theory [65] have been proposed as alternatives to TBT. The advantage of HBTs is providing of the zero transverse shear stress and strain situation at the top and bottom surfaces of the beam without any shear correction factors. Recent times, various microstructure-dependent beam models based on HBTs have been introduced in conjunction with nonlocal elasticity theory [66–74], modified couple stress and strain gradient theories [75–80].

One of the objectives of this paper is to develop a shear deformation beam model for size-dependent nonhomogeneous microbeams. The other one is to propose new shear correction factors including additional coefficients in TBT for analysis of microbeams. The governing equations and corresponding boundary conditions in bending and buckling are obtained by implementing of minimum total potential energy principle. Bending and buckling problems of a simply supported FGM microbeam are analytically solved by Navier solution procedure. The newly obtained results are compared with the results of other beam theories as EBT and TBT and other classical and modified couple stress models. A detailed parametric study is performed to show on influences of gradient index, thickness-to-material length scale parameter ratio (or vice versa), slenderness ratio and shear correction factors.

2. Theory and formulation

The modified strain gradient elasticity theory was proposed by Lam et al. [17] includes higher-order deformation gradients besides classical strain tensor. They can be identified by the Lamé constants and three additional material length scale parameters. The total strain energy density w_i can be expressed for linear elastic isotropic materials as [17,81]

$$w_i = \frac{1}{2}\lambda\varepsilon_{ii}\varepsilon_{jj} + \mu\left(\varepsilon_{ij}\varepsilon_{ij} + l_0^2\gamma_i\gamma_i + l_1^2\eta_{ijk}^{(1)}\eta_{ijk}^{(1)} + l_2^2\chi_{ij}^s\chi_{ij}^s\right) \tag{1}$$

in which $\varepsilon_{ij}, \gamma_i, \eta_{ijk}^{(1)}$ and χ_{ij}^s denote the components of the strain tensor ε, the dilatation gradient vector γ, the deviatoric stretch gradient tensor $\boldsymbol{\eta}^{(1)}$ and the symmetric rotation gradient tensor $\boldsymbol{\chi}^s$, respectively and are defined as [17]

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

$$\gamma_i = \varepsilon_{mm,i} \tag{3}$$

$$\eta_{ijk}^{(1)} = \frac{1}{3}(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}) - \frac{1}{15}\left[\delta_{ij}(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}) + \delta_{jk}(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}) + \delta_{ki}(\varepsilon_{mm,j} + 2\varepsilon_{mj,m})\right] \tag{4}$$

$$\chi_{ij}^s = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}) \tag{5}$$

$$\theta_i = \frac{1}{2}e_{ijk}u_{k,j} \tag{6}$$

where u_i represents the components of displacement vector $\mathbf{u}$ and θ_i represents the components of rotation vector $\boldsymbol{\theta}$, also δ and e_{ijk} are the Kronecker delta and alternating symbols, respectively. In addition, l_0, l_1, l_2 are additional material length scale parameters related to dilatation gradients, deviatoric stretch gradients and rotation gradients, respectively. Also, λ and μ are the Lamé constants as

$$\lambda = \frac{E\upsilon}{(1+\upsilon)(1-2\upsilon)}, \quad \mu = \frac{E}{2(1+\upsilon)} \tag{7}$$

The components of Cauchy (classical) stress tensor $\boldsymbol{\sigma}$ and higher-order stress tensors $\mathbf{p}$, $\boldsymbol{\tau}^{(1)}$ and $\mathbf{m}^s$ are respectively conjugated to ε, γ, $\boldsymbol{\eta}^{(1)}$ and $\boldsymbol{\chi}^s$ can be defined by taking derivatives of the total strain energy density with respect to work-conjugated strains as

$$\sigma_{ij} = \frac{\partial w_i}{\partial \varepsilon_{ij}} = \lambda\delta_{ij}\varepsilon_{mm} + 2\mu\varepsilon_{ij} \tag{8}$$

$$p_i = \frac{\partial w_i}{\partial \gamma_i} = 2\mu l_0^2\gamma_i \tag{9}$$

$$\tau_{ijk}^{(1)} = \frac{\partial w_i}{\partial \eta_{ijk}^{(1)}} = 2\mu l_1^2\eta_{ijk}^{(1)} \tag{10}$$

$$m_{ij}^s = \frac{\partial w_i}{\partial \chi_{ij}^s} = 2\mu l_2^2\chi_{ij}^s \tag{11}$$

According to modified strain gradient elasticity theory, the strain energy U can be written by [17]

$$U = \frac{1}{2}\int_0^L \int_A \left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau_{ijk}^{(1)}\eta_{ijk}^{(1)} + m_{ij}^s\chi_{ij}^s\right) dA dx \tag{12}$$

The displacement components of an initially straight beam on the basis of sinusoidal beam theory (see Fig. 1) can be written as [62]

$$u_1(x,z) = u(x) - z\frac{dw(x)}{dx} + R(z)\left[\frac{dw(x)}{dx} - \varphi(x)\right],$$
$$u_2(x,z) = 0,$$
$$u_3(x,z) = w(x) \tag{13}$$

where u_1,u_2 and u_3 are the x-, y- and z-components of the displacement vector, and also u and w are the axial and transverse displacements, φ is the angle of rotation of the cross-sections about y-axis of any point on the mid-plane of the beam, respectively. $R(z)$ is a

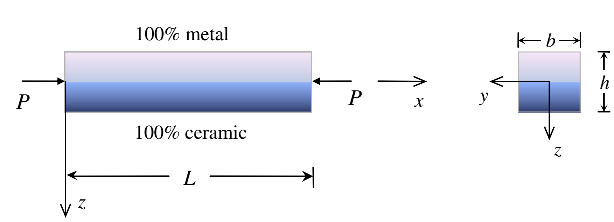

Fig. 1. Geometry of an axially loaded functionally graded microbeam.