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Analysis of functionally graded stiffened plates based on FSDT utilizing reproducing kernel particle method

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ABSTRACT

Using reproducing kernel particle method (RKPM), concentrically and eccentrically functionally graded stiffened plates (FGSPs) are analyzed based on first order shear deformation theory (FSDT). The plates are subjected to uniformly distributed loads with simply supported and clamped boundary conditions. The interactions between the plate and stiffeners are imposed by compatibility equations. Metal-ceramic composition is assumed as the functionally graded material (FGM). Material properties vary through the thickness direction according to the power law of volume fraction. Mori–Tanaka scheme is used to obtain effective material properties. Poisson's ratios of plates and stiffeners are taken to be constant. Full transformation approach is used to enforce essential boundary conditions. Effects of eccentricity of the stiffeners, dimensionless support domain parameter, dimensionless thickness, boundary conditions and the volume fractions of the constituents on the behavior of the stiffened plates are investigated.

1. Introduction

Stiffened plates are used in several engineering applications to improve efficiency of the structures in terms of strength/weight. The application areas of stiffened plates are in aeronautical and mechanical engineering such as aerospace structures, road bridges, and ship hulls.

The earliest models used to simulate the behavior of stiffened plates include grillage model [1] and orthotropic model [2], which were not usable in general problems. Subsequently, models incorporating new concept have been developed which treat the plate and stiffeners as standalone structures and compatibility equations are introduced to consider interactions between the plate and stiffeners [3–5].

Various methods used to analyze the stiffened plate problems are include Rayleigh–Ritz method [6,7], finite difference method (FDM) [8], finite element method (FEM) [5,9–13], constraint method [4], semi analytical finite difference method [14] and finite strip method [15]. Wen et al. used boundary element method to analyze the shear deformable stiffened plates. They used coupled boundary element formulation of shear deformable plate and two-dimensional plane stress elasticity [16]. Peng et al. [3] analyzed rectangular stiffened plates based on first order shear deformation theory (FSDT) and element-free Galerkin (EFG) method. They imposed compatibility conditions between the plate and stiffeners for both concentric and eccentric stiffeners.

Numerical methods are essential for simulating stiffened plates. Among them FEM gained considerable attentions because of its efficiency and stability. However, the shortcoming of FEM is its inherent dependency on mesh. To eliminate this drawback the idea of well-known meshfree or meshless method that relies only on nodes is introduced [17,18].

Belytschko et al. [19] proposed the EFG method subsequently used to analyze thin plates [20], thin shells [21] and solids [19,22].

Smoothed particle hydrodynamics (SPH) is also one of the meshfree methods considered as a kernel-based interpolation method first used for modeling astrophysical phenomena [23,24]. Liu et al. [25,26] found that the SPH approach has considerable weakness in satisfying consistency conditions specially on boundaries. The stability of SPH is discussed by Aluru [27]. The consistency of SPH can be established by introducing a correction function. This approach was proposed by Liu et al. [25,26] and named it as reproducing kernel particle method (RKPM). RKPM has been used for solving several linear and non-linear problems in mechanics [26,28–30]. This method has also been used for analysis of composite plates [31] and functionally graded plates [32] which shows good accuracy and stability characteristics in solving solid mechanics problems.

The essential boundary conditions cannot be directly imposed in RKPM method due to the fact that RKPM shape functions do

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not satisfy the Kronecker delta property. Several approaches are used to impose boundary conditions including Lagrangian multiplier method [19], penalty method [33], and full transformation method [34]. In addition, Wang et al. [35] proposed special technique to directly impose essential boundary conditions for EFG method using local Kronecker delta property of the moving least square (MLS) approximation.

Functionally graded materials (FGMs) made up of two constituents e.g. metal and ceramic are widely used. The composition is varied continuously along certain directions according to volume fraction from a ceramic-rich surface to a metal-rich surface [36].

Because of unique features of FGMs like their high gradient temperature resistance, they can be used in wide range of applications such as spacecraft structures. Furthermore, in an FGM, material properties can be customized to optimize the desired characteristics, e.g. minimizing the maximum deflection of the plates [37].

In recent years, many investigations have been done in the field of functionally graded plates (FGPs). Vel and Batra [38,39] presented an exact solution for thermoelastic deformations and vibration of FGPs. Qian et al. analyzed elastic and thermoelastic deformations of FG plates. They employed the meshless local Petrov–Galerkin (MLPG) method to solve the governing equations of plates [40–42]. The bending and free vibration analyses of FGPs based on FSDT were presented by Thai and Choi [43]. Gilhooly et al. studied thick FGPs based on high-order shear and normal deformation plate theory using MLPG method [44]. A thorough review of meshless methods including EFG and RKPM for laminated and functionally graded plates and shells can be found in [45]. Moreover, a critical review of recent researches on FGPs were presented in [46].

In addition, FEM is widely used to analyze FGPs pioneered by Reddy [36]. He proposed a general formulation for FGPs using the third-order shear deformation plate theory and developed its associated finite element (FE) model. Naghdabadi and Hosseini proposed an FE formulation to analyze FG plates and shells [47]. Valizadeh et al. developed a NURBS-based finite element for static bending, vibration, buckling and flutter analyses of FGPs [48]. The static and vibration analyses of FGPs were presented by Ferreira et al. using collocation method employing shear deformation theories [49,50]. Zhang and Zhou proposed a formulation to analyze the FGPs based on physical neutral surface of the plate [51]. Moreover, the influence of neutral surface position on the non-linear behavior of FGPs were investigated by [52,53].

Ray and Sachade [54] suggested an FE model for the static analysis of FG plates integrated with a layer of piezoelectric fiber reinforced composite (PFRC) material. They investigated the effect of varying piezoelectric fiber angle in the PFRC layer on its actuating capability of the FG plates.

In this paper, the functionally graded stiffened plates (FGSPs) is analyzed based on FSDT using RKPM. Circular and rectangular plates under uniformly distributed loads for both simply supported and clamped boundary conditions are investigated. A metal-ceramic composition is assumed as the FGM in which mechanical properties vary only along the thickness direction and the Poisson's ratio is considered to be constant. The principle of minimum potential energy is employed to obtain Galerkin weak-form formulation of the FGSPs. The neutral-surface based formulation is used and the exact shear correction factor is applied. 1D- and 2D- RKPM shape functions are utilized to approximate deformation fields of stiffeners and the plate, respectively. The RKPM is used to explore its capability to analyze stiffened plates, which is an extension for development of numerical studies on functionally graded structures. The full transformation method applied in this study can be achieved by reconstruction of RKPM shape functions which possess Kronecker delta property, in this way boundary conditions can be imposed directly. The interactions between the plate and stiffeners are imposed by compatibility equations in which deflections of stiffeners are considered to be equal to that of the plate along the line of interaction.

2. Functionally graded material

Consider a functionally graded plate composed of ceramic and metal phases. The material on the top surface of the plate is ceramic and is graded to metal at the bottom surface of the plate by the power law distribution. The volume fractions of ceramic V_c and metal V_m are given by

$$V_c = \left(\frac{1}{2} + \frac{z'}{h}\right)^n, \quad V_m = 1 - V_c, \tag{1}$$

where z' is the thickness coordinate ($-h/2 \leqslant z' \leqslant h/2$), and n is the material constant. Fig. 1 depicts the through-the-thickness variation of the ceramic phase volume fraction for different values of n.

The homogenized material properties are evaluated using Mori–Tanaka formulation [55]. According to the Mori–Tanaka scheme the effective elastic properties of the FGM are given by

$$\begin{aligned} \frac{K - K_c}{K_m - K_c} &= \frac{V_m}{1 + (1 - V_m)\frac{3(K_m - K_c)}{3K_c + 4G_c}}, \\ \frac{G - G_c}{G_m - G_c} &= \frac{V_m}{1 + (1 - V_m)\frac{G_m - G_c}{G_c + f_c}}, \end{aligned} \tag{2}$$

where

$$f_c = \frac{G_c(9K_c + 8G_c)}{6(K_c + 2G_c)}, \tag{3}$$

in which K and G are bulk modulus and shear modulus, respectively. The subscript c and m refer to ceramic and metal phases, respectively. K and G are related to Young's modules, E, and Poisson's ratio, v, by the following equations

$$E = \frac{9KG}{3K + G}, \quad v = \frac{3K - 2G}{2(3K + G)}. \tag{4}$$

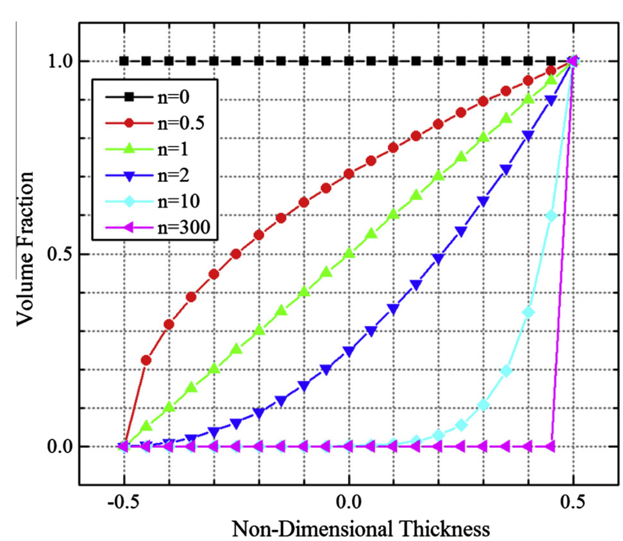

Fig. 1. Through-the-thickness variation of the ceramic phase volume fraction for different values of n.