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First strain gradient elasticity solution for nanotube-reinforced matrix problem

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ABSTRACT

In the current paper, a rigorous proof of an important theorem which has been used frequently for derivation of field equations of first gradient elasticity is given for the first time. After that, due to the wide use of nanoparticles as reinforcements to different types of matrices, the nanotube-reinforced matrix problem is investigated in cylindrical coordinates. Then, using the most general model for an isotropic gradient elastic material, the displacement formulation is employed to solve the governing equations of the nanotube-reinforced matrix problem. For this purpose, the generalized perfect interface conditions for the nonhomogeneous representative volume element (RVE) are introduced and used to derive the solution. Numerical results reveal that as the matrix characteristic length parameter becomes larger in comparison to that of nanotube, the difference between the results of the classical theory and the strain gradient theory will increase and classical theory cannot accurately predict the mechanical response of the RVE; In addition, increasing the nanotube's volume fraction results in reduction of the maximum compressive stress and a rise in the overall stiffness of RVE.

1. Introduction

Due to the growth of nanoscience and nanotechnology, during the past two decades new branches of engineering, medicine and science have been developed. Ijima's discovery [1] of carbon nanotubes (NTs) with unique electrical and mechanical properties made them one of the best modern materials used in the field of nanotechnology [2]. Compared with conventional polymer composites, nanotube-reinforced polymer composites have significantly increased strength and stiffness [3].

In order to determine the mechanical behavior of nanomaterials, there are two main approaches: (1) Experimental methods and (2) Mathematical modeling. Since the experimental methods are expensive and difficult to apply at the nanoscale, the second approach is used more, which is divided into the following categories: (a) atomistic modeling, (b) hybrid molecular-structural mechanics and (c) continuum mechanics. Simulating large scale atomic systems using the first two methods cost much more time in comparison with that of continuum mechanics; furthermore, formulation of former methods is more complex than the latter one. Thus, it indicates that the continuum mechanics can be used as an alternative solution to inspect mechanical behavior at nanoscale.

The inability of classical continuum mechanics in explaining the size effects observed in very small scales led to the development of generalized theories such as micropolar, micromorphic, couple stress, nonlocal and strain gradient theory. The aforementioned theories were used by researchers to study the elastic and plastic deformation in different areas such as micropolar plasticity in crystals [4]. Among these theories, strain gradient theory is a theory which, in addition to the strain, gradients of strain must be considered in the strain energy density function of the deformable body. The beginning of investigation into the strain gradient theory can be found in studies of Mindlin [5] and Kröner [6]. After 1960s, many studies on the Mindlin's general strain gradient theory were carried out. In the early 1990s, Aifantis and his coworkers [7] based on previous studies in the field of plasticity [8] and nonlinear elasticity [9] proposed a special case of the strain gradient theory where strain and also Laplacian of strain were observed in constitutive equations. Compared with models in [5,6], the Aifantis model (simplified strain gradient theory) [7] is of more practical importance. This stems from its simpler implementation not only because just one internal length is involved, but also because of the structure of the governing equations which is dominated by the Laplacian – an operator that has been studied extensively in many scientific fields [10]. Also, a more general form of these Laplacian-based constitutive equations are studied in detail by Aifantis [11]. Furthermore, Askes and Gitman [12] compared the Aifantis model and Eringen's nonlocal theory [13] and concluded

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that Aifantis theory is more appropriate in FEM analysis. This is due to the Laplacian-dominating structure of the governing equations and the ability to use the so-called Ru-Aifantis theorem [14]. In recent years, researchers used Aifantis model to study Kirchhoff microplate [15], static deformation of an elastic circular micro-plate [16], half-space and half-plane contact problems [17] and the size-dependent behavior of bilayered microbeams [18].

Yang et al. [19] developed the modified couple stress theory based on strain gradient theory. In this way, strain tensor and rotation gradient were the only factors contributing to the strain energy density function. They also provided modified constitutive equation for couple stress theory in which only a new length scale parameter can be seen. Also, modified couple stress theory has been employed to investigate the thermal effect on buckling and free vibration behavior of functionally graded microbeams [20], microstructure-dependent behavior of laminated composite Timoshenko beam [21] and size-dependent buckling behavior of functionally graded microbeam in an elastic medium [22]. As stated above based on this theory, in addition to strain, rotation gradient contributes to deformation. Hence, if the curl of displacement field vanishes in special cases, e.g. extension of microbar, based on this theory, the size effect cannot be captured. Lam et al. [23] presented a new modified strain gradient theory based on Mindlin's strain gradient theory. They introduced three new length scale parameters for the material, as well. Experimental tests on epoxy beam were performed to extract characteristic length parameters. In this theory, the asymmetric part of curvature tensor is not the only factor which has an effect on the deformation of the body. In addition, the symmetric part is involved. Mohammadi and Mahzoon [24] investigated the thermal effects on postbuckling of nonlinear microbeams and Lam et al. model [23] was employed to capture size effects. Based on an earlier study [23], Wang and Lam [25] employed a particular format of strain gradient elasticity in which the only contributing higher-order term is rotation gradient leading to the so called couple stress theory. Askes and Aifantis [26] showed that the claim of Wang and Lam that the classical solution can be used as a substitute for gradient solution is not valid in general. They investigated the shear layer problem to prove this issue and obtained that the difference between classical and gradient solution depends on the size of specimen. Also, they suggested the well-known methodology of Ru and Aifantis [14] to implement the classical solution as a surrogate for gradient solution. Furthermore, they concluded that for applying the aforementioned methodology, the rotational gradient should be accompanied with dilatational gradients or deviatoric stretch gradients.

There are various studies on strain gradient theory in order to investigate the size effect at nanoscale. Some of studies in this theory on structures such as bars, beams, and shells are used to investigate the static and dynamic behavior of nanostructures like carbon NTs. Askes and Aifantis [27] have studied bending wave dispersion in NTs, using the Aifantis model. For this purpose they have used Euler – Bernoulli and Timoshenko beam models and compared their results with molecular dynamics simulations. They have concluded that the instability of wave frequency can be removed if the inertia gradient is considered in the governing equations of the strain gradient theory. Wang [28], using strain gradient theory and taking into account the effect of inertia gradient, was able to study wave propagation in fluid-carrying NTs. Wang et al. [29] examined the mechanical and thermal buckling of NTs placed in the elastic matrix. For this purpose, Euler – Bernoulli and Timoshenko beam theories have been employed and the results indicate that the critical buckling load depends on the size effects and elastic matrix's stiffness. To investigate the vibrational behavior of single-walled NTs, Ansari et al. [30] have used Euler – Bernoulli and Timoshenko beam models based on strain gradient theory. The results signify that as the NT aspect ratio is reduced to about 6, the differences between the various gradient theories will be greater and strain gradient theory with the effects of inertia gradient gives better results which are consistent with the results of molecular dynamics simulations.

In addition to the aforementioned studies, researchers used the classical problems of thick-walled cylinder and sphere under pressure in the cylindrical and spherical coordinates to conduct a series of studies on nanostructures. Collin et al. [31] studied the static responses of thick-walled cylinder problem affected by the changes in special sets of strain gradient parameters appeared in boundary conditions. On the other hand, Gao and Park [32] presented required relations for thick-walled cylinder under pressure considering the Aifantis model and aforementioned problem was inspected. It was observed that when the material length scale is comparable to the radius of the cylinder, size effects are dominant. Gao et al. [33] also investigated the problem of thick-walled sphere employing the Aifantis model and the obtained results were similar to previous work [32]. Considering the surface effects, Zheng et al. [34] have studied the NT under pressure using a simplified strain gradient theory. They have concluded that the effects of the length scale parameter compared to those of directional surface energy length scale parameter on stress distribution of the NT are different. When length scale parameter decreases, the stress distribution will be closer to the classical theory; however, by increasing the length scale of the surface energy parameter, the results tend to the classical theory.

In recent years, solving the problem of inhomogeneity in gradient elasticity has attracted the researchers' interest. Among them, Gao and Ma [35] formulated the Eshelby problem of infinite homogeneous isotropic elastic material containing an inhomogeneity using the Aifantis model. They derived a new Eshelby tensor which consists of a classical part and a gradient part. Also, they obtained the components of the new Eshelby tensor for a spherical inhomogeneity and showed that when the inhomogeneity radius is small, the effect of the gradient part is large and cannot be ignored. Gao and Ma [36], extending the previous work, obtained a solution for the Eshelby problem of finite homogeneous isotropic elastic spherical matrix containing an concentric spherical inhomogeneity using the Aifantis model. Also, they showed that the finite Eshelby tensor [36], compared with the infinite Eshelby tensor [35], depends on not only the position, inhomogeneity size and material length scale parameter, but also the matrix size. As can be seen in the sequel, the present work can also be considered as solving an inhomogeneity problem in gradient elasticity.

Considering that the NTs have been used as reinforcement in some applications of polymer nanocomposites; it is very important that a volume element consisting of a NT surrounded by polymeric matrix be scrutinized to determine the stress distribution in each component. This makes it possible to investigate the NT load-carrying capability in polymer reinforcing. Since the nanoscale is discussed, considering the size effect as a factor in determining the behavior of nanomaterials is important.

To the best of authors' knowledge, the only research on core/shell nanowire was conducted by Aifantis et al. [37] based on strain gradient plasticity and no research has been carried out on NTs surrounded by a matrix using strain gradient elasticity theory. The aim of this study is to consider the effect of microstructural and geometrical characteristics on the mechanical behavior of the representative volume element (RVE). Through a review on derivation of field equations, an important theorem which plays a fundamental role in deriving the field equations is proven for the first time (see Appendix A). Then, using the energy density function purposed by Mindlin [5], response of Navier equations was extracted for the NT-reinforced matrix by employing the solution of thick-walled cylinder problem. After that, considering perfect interface assumption, boundary conditions are extracted for the aforementioned problem. At the end, numerical results are presented and microstructural effects are discussed.