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A layer-wise theory for laminated glass and photovoltaic panels ☆

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ABSTRACT

Laminated plates with glass skin layers and a core layer from soft polymers are widely used in the civil engineering. Photovoltaic panels currently available on the market are composed from stiff front and back layers and a solar cell layer embedded in a soft polymeric encapsulant. In this paper a layer-wise theory for the structural analysis of glass and photovoltaic laminates is developed. Starting from governing equations for individual layers, kinematical constraints and appropriate interaction forces, a twelfth order system of partial differential equations is derived. The primary variables in the theory include the Airy stress function, the deflection function and the vector of relative in-plane displacements of skin layers. For symmetric laminates a system of uncoupled differential equations with respect to scalar potentials is presented. Three of them correspond to the first order shear deformation plate. The new additional second order differential equation provides a correction function according to the layer-wise theory. Closed form analytical solutions for a plate strip are derived to illustrate the essential influence of this correction for laminates with soft core layer. The importance of additional boundary conditions is shown for examples of free and framed plate edges.

1. Introduction

Laminated plates with glass skin layers and a core layer from Polyvinyl Butyral (PVB) are widely used in the civil engineering and automotive industry [1–3]. Crystalline or thin film photovoltaic modules currently available on the market are composed from front and back glass or polymer layers and a solar cell layer embedded in a polymeric encapsulant [4–6]. Fig. 1 illustrates basic components of a crystalline silicon solar cell panel. Material like Ethylene Vinyl Acetate (EVA) and PVB are usually applied to encapsulate the solar cells [4]. In new lightweight variants of photovoltaic modules the front and back plates are made from plastics. These skin layers are connected together by a transparent Polyurethane (PUR), in which the solar cells are embedded [7]. Certification procedures of terrestrial crystalline photovoltaic modules are given in the norm [8]. Among various requirements, mechanical tests to simulate wind and snow loads are prescribed. Additionally, solar modules must withstand non-stationary thermal profiles due to daily or season-dependent temperature cycles.

For design of solar modules it is beneficial to analyze the suitability of materials like PVB, EVA or PUR for embedding solar cells. These encapsulates have to compensate different mechanical and thermal strains of bottom and top layers. Delaminations between the layers are not allowed and solar cells have to be protected against oxygen and water. Mechanical properties of soft encapsulate materials are usually affected by the manufacturing process. Furthermore, environmental effects can lead to changes in mechanical behavior over time. Therefore a reliable assessment of the stiffness properties is only possible by the testing of a prototype, e.g. by the bending testing of a beam or a plate. To evaluate the test results robust relationships between the applied load, the deflection as well as the transverse shear stress/strain of the encapsulant layer are desirable. Furthermore, such relationships are useful in design of photovoltaic modules.

One feature of laminated glass plates or laminates used in photovoltaic industry is the layered composite with relatively stiff skin layers and relatively thin and compliant polymer encapsulant layer. Let G_s be the shear modulus of the glass skin layer and G_c the shear modulus of the polymeric core layer. The ratio of the shear moduli $\mu = G_c/G_s$ for materials used in photovoltaics is in the range between 10^{-5} and 10^{-2} , depending on the type of polymer and the temperature [4,7,9]. For classical sandwich applications this ratio is in the range of 10^{-2} and 10^{-1}. In addition, in classical sandwich structures the face sheets are thin in comparison with the core, while in photovoltaic applications the face layers are relatively thick and the core is relatively thin.

☆ Dedicated to Prof. Dr.-Ing. habil. Dr. h. c. Johannes Altenbach on the occasion of his 80th birthday

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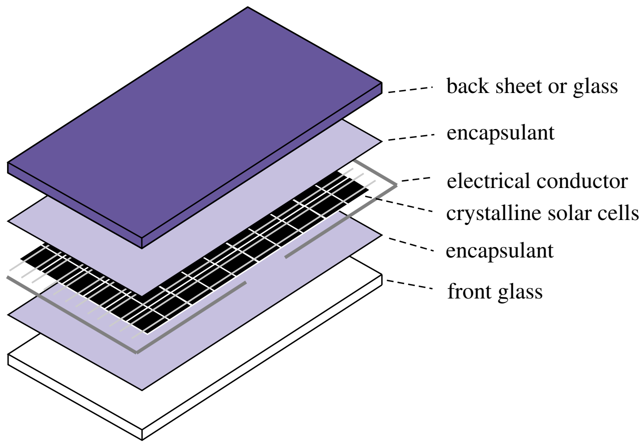

Fig. 1. Components of crystalline solar modules.

To analyze the behavior of laminated plates various structural mechanics models are available. A widely used approach for sandwich and laminate structures is the first order shear deformation theory (FSDT) [10,11]. The principal assumption of this theory is that the normals to the midsurface of a plate behave like rigid bodies during the deformation. The mechanical interactions between the cross sections is characterized by forces and moments. The advantage of this theory is the possibility to solve the governing differential equations in a closed analytical form for plates of various shapes. Closed form solutions or approximate analytical solutions for plates according to the FSDT are presented in [10–16], among others. A problem related to FSDT is to estimate effective characteristics of the layered system, in particular the properties related to the transverse shear deformation. Closed form relationships are developed to find effective elastic stiffness of a laminate from the properties of layers, e.g. [4]. However, numerical techniques are required to estimate the effective transverse shear deformation in the inelastic range, [17,18]. Furthermore, for laminates with extreme differences in the stiffness properties of constituents the FSDT fails to predict the deformation properties of the laminate correctly, as shown for example in [4,7] for beams.

Laminated glass and photovoltaic panels can also be analyzed by the use of three-dimensional theory of elasticity and applying the finite element method for the numerical solution. To this end various types of continuum shell finite elements and three-dimensional solid finite elements are available in commercial codes, e.g. [19]. Due to extreme differences in material properties of constituents and the relatively low thickness of the core layer considerable numerical effort is required to obtain the results with a desired accuracy [4,20]. In particular, care should be taken for finite element meshing the core layer in order to compute the transverse shear strains and the related stresses accurately.

Recently zig-zag and layer-wise theories were developed and applied to analyze laminated structures. A zig-zag theory approximates the displacements by piecewise functions with respect to the thickness coordinate such that the compatibility between the layers is fulfilled. Then the governing equations of the three-dimensional elasticity theory are reduced to the two-dimensional plate equations by means of variational methods or asymptotic techniques [21–23]. Within the layer-wise theory (LWT) balance and constitutive equations are derived for individual layers. With constitutive assumptions for interaction forces and compatibility conditions a model for the layered system is derived. For laminated beams with core layer from soft polymers LWT are presented in [1–4,7], among others. To derive the robust equations the assumption is made that glass skin layers deform according to the Bernoulli–Euler beam theory, i.e. the transverse shear deformations are negligible. The soft core layer carries out the transverse shear stress only, while the bending moment and the normal force are negligible. In [1,4,7] results of three point bending tests for beams with core layers from various polymers are presented. Closed form solutions derived with LWT agree well with the experimental data. Furthermore, as shown in [3,4,7] the solutions according to LWT agree well with the results of the three-dimensional finite element analysis. The LWT was found to be more attractive if compared to the zig-zag theories since the load transfer between the layers can be analyzed explicitly. Furthermore, with proper assumptions about stiffness and/or deformation of individual layers, LWT can provide equations that are easier in comparison to the zig-zag theories, and can be solved in a closed analytical form. Despite the fact that LWT was found efficient, the majority of publications deal only with beam equations. Recently a LWT for laminated glass plates is proposed [24]. The deformation of skin layers is described by the Kirchhoff plate theory, while the core is modeled as the shear layer, that is, the membrane and bending states are ignored.

The aim of this paper is to derive a theory for the use in analysis and design of glass and photovoltaic panels. In addition to the previous work we address the following problems.

- For photovoltaic panels several extensions to the available theories of beams [3,4,7] and plates [24] are required. Indeed, solar cells do not contribute essentially to the overall bending and membrane stiffness of the laminate. Therefore, for the global deformation analysis the core layer including the polymer encapsulant with embedded solar cells, can be considered as homogenized "shear" layer. This is consistent with the available theories for laminated glass [3,24]. However, for photovoltaic panels robust relationships are required to compute the local loading/deformation exerted on the solar cells from the global characteristics of the laminate.
- For laminated glass beams/plates only the lateral forces and deformations are analyzed [3,24]. Solar panels are usually positioned at a certain angle to the horizontal. Therefore, mechanical loads like the panel or snow weight produce the tangential force components acting on the laminate. This requires to account for in-plane stress/deformation state as well as additional shear stress/deformation of the laminate.
- The edges of photovoltaic laminates are usually fixed by frames to restrict the relative sliding of skin layers. To analyze the influence of frames on the global behavior of the panel appropriate boundary conditions are required.

2. Layer-wise theory

Fig. 2 shows a sketch of a rectangular three-layered plate. The Cartesian base vectors $\mathbf{i}_1,\ \mathbf{i}_2,\ \mathbf{n}$ and the corresponding coordinates $x_1,\ x_2$ and z are used to specify the position vectors in the reference state. l_1 and l_2 designate the length and the width of the plate while $h_T,\ h_C$, and h_B denote the thicknesses of the top, core and bottom layers, respectively. In what follows all quantities related to the top, core and bottom layers, will be denoted by subscripts $T,\ C$, and B, respectively. The origin for the z coordinate is placed in the midplane of the core layer as shown in Fig. 2, such that $-h_B - h_C/2 \leqslant z \leqslant h_C/2 + h_T$. In this Section we present basic equations for the individual layers. They include the equilibrium conditions, the constitutive equations, the compatibility conditions for strains in layers and kinematical constraints between the layers. Finally we make assumptions to simplify the governing equations.

Below we apply the direct tensor calculus in the sense of Gibbs [25] and Lagally [26]. For basic rules of the direct tensor notation one may also consult [27,28], among others. The Greek indices take values 1 and 2 and the Einstein summation convention over repeated indices will be used.