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Nonlinear thermal stability and vibration of pre/post-buckled temperature- and microstructure-dependent functionally graded beams resting on elastic foundation

M. Komijani [a], S.E. Esfahani [b], J.N. Reddy [c], Y.P. Liu [d,*], M.R. Eslami [a]

[a] Mechanical Engineering Department, Amirkabir University of Technology, Tehran 15914, Iran
[b] Mechanical Engineering Department, Islamic Azad University, South Tehran Branch, Tehran, Iran
[c] Department of Mechanical Engineering, Texas A&M University, College Station, TX 77843-3123, USA
[d] School of Civil Engineering and Transportation, South China University of Technology, Guangzhou 510640, China

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ABSTRACT

In this study, buckling and post-buckling analysis and small amplitude vibrations in the pre/post-buckling regimes of functionally graded beams resting on a nonlinear elastic foundation and subjected to in-plane thermal loads are investigated. The Timoshenko beam theory with the von Kármán nonlinearity and the microstructural length scale based on the modified couple stress theory are used to derive the governing nonlinear equilibrium equations. Various types of boundary conditions are considered. Thermo-mechanical properties of the FGM beams are assumed to be functions of both temperature and thickness. The solution is determined in two different regimes. A static phase with large amplitude response and a dynamic regime near the static one with small amplitudes are considered. In order to discretize the motion equations in geometrical domain of both regimes, generalized differential quadrature method (GDQM) is used. The resulting system of nonlinear algebraic equations are solved iteratively using Newton's method. Numerical results indicate that, depending on the boundary conditions and the type of thermal load, the response of the structure may be of unique stable path or the bifurcation-type in static regime. Also, free vibration of a beam subjected to in-plane thermal load may show zero frequency magnitude at a certain temperature, which specifies the existence of bifurcation-type of instability. Influences of nonlinear elastic foundation parameters, thermal load type, different types of boundary conditions, and microstructural length scale on equilibrium paths, critical buckling load, and fundamental frequencies are studied.

1. Introduction

Thermal buckling is a phenomenon that should be checked for the stability of structural design problems. Consequently, thermal stability and vibration analysis of beam structures with or without elastic foundation are common in structural mechanics. Li et al. [1] studied buckling and post-buckling regimes of elastic rods under thermal loads. They obtained numerical results by solving the nonlinear equilibrium equations of slender Euler–Bernoulli pinned–fixed beams via the shooting method. Li et al. [2] employed the shooting method to unravel the equations related to buckling and post-buckling behavior of fixed–fixed elastic beam under transversally applied nonuniform temperature load. Very recently, Komijani and his co-workers [3] proposed an analytical approach to investigate the thermal stability analysis of two–layer Timoshenko beams with partial interactions and interlayer slips. They concluded that the shear bond stiffness of the continuous shear connectors between two layers has noticeable effect on the critical buckling temperature of the composite structure. Thermal stability analysis of the Euler–Bernoulli beams placed on a two-parameters nonlinear elastic foundation is investigated by Li and Batra [4]. This work revealed that the Winkler foundation possesses capability on mode periodicity of buckling configuration of a pinned–fixed and pinned–pinned beams. Material properties are assumed to be temperature independent in this works.

Bhangale and Ganesan [5] utilized finite element method to examine the buckling and vibration of an FGM sandwich beam with constrained viscoelastic layer in thermal environment. They investigated the effect of temperature on natural frequency and

* Corresponding author. Tel.: +86 13318889728; fax: +86 20 87114460.
E-mail address: tcypliu@scut.edu.cn (Y.P. Liu).

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loss factors for the considered beam. Finite element method is applied based on the classical laminate plate theory by Ramkumar and Ganesan [6] to study vibration and buckling of FGM thin-walled box columns in thermal environments. Moreover, material properties are presumed to be temperature-dependent. Zhao et al. [7] investigated post-buckling for FGM rod with both ends pinned subjected to thermal load by using shooting method to solve the obtained equations. Study on post-buckling behavior of functionally graded material beams including an edge crack effect based upon assumptions involving von-Karman's strain–displacement relations combined with the Timoshenko beam theory along with an open edge crack is reported by Ke et al. [8]. They used the Ritz method to obtain the nonlinear governing equations and applied the Newton–Raphson method to linearize the discreted equations. In order to obtain the buckling and thermal post-buckling governing equations of uniform slender FGM beams, the von-Karman strain–displacement relationship is used by Anandrao et al. [9]. Single-term Ritz method and finite element method are utilized to represent the response of the beam. Ma and Lee [10] applied shooting method to show the response of FGM beams with simply-supported edges under in-plane thermal loading. Furthermore, in this research material properties have been considered to be temperature-dependent. Ma and Lee [11] have studied the nonlinear static responses of FGM beams under in-plane thermal loading. They obtained an analytical solution for the response of clamped–clamped as well as pinned–pinned beams subjected to uniform temperature rise loading.

Li et al. [12] reported numerical results using the shooting method for nonlinear free vibrations and thermal post-buckling analysis of a hybrid FGM Timoshenko beam with both ends clamped. Wang et al. [13] analyzed thermal buckling, post-buckling, and nonlinear vibrations in both pre and post bucked regimes on the basis of an exact analytical approach and the physical neutral surface concept for an FGM Euler–Bernoulli beam bonded with and without piezoelectric layers. They assumed that the material properties are temperature dependent and confined their study to clamped–clamped boundary conditions. The free vibration of FGM sandwich beam including thickness variations in thermal environment is studied by Pradhan and Murmu [14]. Xiang and Yang [15] reported the influence of the temperature variation through the beam's height on small free vibrations of symmetrically laminated FGM beams base on the Timoshenko beam theory. More recently, Komijani et al. [16,17] investigated the nonlinear thermal stability and small amplitude vibrations of a piezoelectric beam with graded properties subjected to thermo-electrical loadings. It is demonstrated that since the material distribution of the smart FGM beam is not symmetric with respect to the beam mid-surface, the linear bifurcation buckling may not occur in this type of beams. Small amplitude vibrations of beams subjected to in-plane mechanical or thermal loads are studied using different solution methods to solve the resulting governing equations; see, for example, finite element method [17], shooting method solution [12], differential quadrature method [14,45], variational iteration method (VIM) solution [42], single-term Galerkin solution [13], and the exact solution [46].

The conventional continuum mechanics theories do not have ability to explain the size effects in micron- and nano-scale structures. Nonclassical continuum theories such as the modified couple stress theory and higher-order gradient elasticity theories are reasonably capable of determining the microstructural length scales effects. The couple stress theory introduced by Koiter, Mindlin and Toupin [18–20] was modified by Yang et al. [21] to have a single length scale parameter. Using the modified couple stress theory of Yang et al. [21], Ma, Gao, and Reddy [22–24], Reddy [25], Liu and Reddy [26], Reddy and Kim [27], Arbind and Reddy [28], Arbind, Reddy, and Srinivasa [29], and Kim and Reddy [30] developed solutions for bending and vibration of Timoshenko and Reddy beam theories and classical and shear deformation plate theories. Recently, Srinivasa and Reddy [31] developed a finite deformation gradient elasticity theory for a fully constrained finitely deforming hyperelastic Cosserat continuum where the directors are constrained to rotate with the body rotation. This is a generalization of small deformation couple stress theories of Koiter, Mindlin and Toupin [18–20]. Such a theory is useful, for example, in modeling an elastic material with embedded stiff short fibers or inclusions, that is, materials with carbon nanotubes or nematic elastomers, cellular materials with oriented hard phases, open cell foams, and so on (see also [61]). Very recently, Komijan et al. [32,33] developed a nonlinear FE model to investigate the effect of size dependency on the nonlinear behavior, stability, and free vibration responses of pre/post buckled functionally graded piezoelectric actuators at the onset of load- and deformation-controlled excitations. The effects of load type, length scale parameter, and applied actuator voltage on different aspects of the structure have been demonstrated through various numerical illustrations.

Another class of papers that deal with a microstructural length scale parameter are based on Eringen's nonlocal elasticity [34]. Bending, buckling, and natural vibration of classical and shear deformable beams are examined by Reddy [35,36] and Reddy and Pang [37] with Eringen's [34] nonlocal model, while Ke et al. [38] and Ke and Wang [39] investigated natural vibration problem of a piezoelectric beams involving Eringen's nonlocal effects in the presence of thermo-electro-mechanical fields.

Stability analysis of the FGM beams which are in-contact with an elastic foundation are limited in number. Sahraee and Saidi [40] employed the differential quadrature method to evaluate the buckling and vibration of a deep FGM beam-columns placed on a Pasternak-type elastic foundation. Fallah and Aghdam [41,42] investigated the nonlinear vibration and post buckling behavior of functionally graded material beams on a nonlinear elastic foundation under axial mechanical [42] or thermal [41] forces. Single mode Galerkin-based method is considered to deduce the critical buckling and post-critical state of the beams. In this research, properties are presumed to be temperature independent and the response of structure is restricted to its first mode. Nonetheless, as insisted by Hetenyi [43], the Winkler elastic foundation significantly affects the buckling shape of the beam and thus limiting the buckled-shape of an in-contact beam like its contact-less shape causes the overestimation of both critical buckling temperature and post-buckling shape. Esfahani et al. [44,45] studied nonlinear stability and vibration analysis of temperature-dependent thermally pre/post-buckled FGM beam supported on nonlinear hardening elastic foundations. They implemented the generalized differential quadrature (GDQ) method to specify the response of the structure subjected to uniform temperature rising and heat conduction across the thickness. Due to various numerical investigations, it is revealed that temperature dependency of material properties has a noticeable effect on the nonlinear thermal stability and vibration behavior of the discussed structure.

The review of literature presented here demonstrates that there are not any published works in the literature that evaluated the nonlinear thermal stability and small amplitude free vibration analysis of temperature and microstructure-dependent FGM beams resting on a nonlinear elastic foundation. The present study overcomes this gap in the literature by considering beams subjected to two types of thermal loads, namely, heat conduction across the thickness and uniform temperature rise. Different combinations of clamped and simply-supported edge conditions are used. Properties of the graded beam are assumed to be functions of temperature, as well. The GDQ method is employed to discretize the equations of motion. An examination of the nonlinear response shows that there may exist a bifurcation state, depending on