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Geometrically nonlinear analysis of composite plates and shells via a quadrilateral element with good coarse-mesh accuracy

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ABSTRACT

This paper presents an improved finite element computational model using a flat four-node element with smoothed strains for geometrically nonlinear analysis of composite plate/shell structures. The von-Karman's large deflection theory and the total Lagrangian approach are employed in the formulation of the element to describe small strain geometric nonlinearity with large deformations using the first-order shear deformation theory (FSDT). The element membrane-bending and geometric stiffness matrices are evaluated by integration along the boundary of smoothing elements which can give more accurate numerical integrations even with bad shape distortions. The predictive capability of the present model is demonstrated by comparing the present results with analytical/experimental and other numerical solutions available in the literature. Numerical examples show that the present formulations can prevent loss of accuracy in distorted or coarse meshes, and therefore, are superior to those of other bilinear quadrilateral elements.

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1. Introduction

The extensive use of composite materials in various types of plates and shells is of considerable interests to many researchers in the field of developing simple and efficient plate/shell element for geometrically nonlinear analysis of these structures. The accurate prediction of structural response characteristics in the large deformation regime therefore becomes more important considerations of engineering design.

Geometrically nonlinear analysis is usually performed iteratively for each load increment with subsequent updating of coordinates and internal stresses to obtain the equilibrium in the deformed configuration. As a result, geometrically nonlinear analysis is considered as a complex issue that requires efficient and reliable advanced numerical methods. Numerical methods such as finite element methods have been developed and widely used for nonlinear analysis of these structures with complex geometry and loading history. There is a vast amount of literature on geometrically nonlinear analysis of plates/shells which is impossible to list all here. An excellent review of the development of plate/shell finite elements during the past 20 years was presented by Yang et al. [1]. Further extensive references on plates/shells can be found in a detailed review of references [2,3]. Some recent latest numerical methods have been also developed and achieved remarkable progress in solving plate and shell problems including linear analysis [4–6] and fracture analysis [7–13], for example.

As discussed in many references [14–19], flat elements have been often and widely used owing to the ease to mix them with other types of element, the simplicity in their formulation and the effectiveness in performing computation. Consequently, flat elements are advantageous in solving the geometrically nonlinear problems in which the response of the structure at each increment/iteration needs to be computed and stored with a large number of history variables. A large number of flat four-node element formulations have been presented to date, for example [20,21], showing good performance. However, to the best authors' knowledge, no literature is available for geometrically nonlinear analysis of composite plates/shells when meshes are coarse or elements are highly distorted. Therefore, the present study is undertaken to address this shortcoming.

The goal of this work is to further develop the flat element MISQ20 (Mixed Interpolation Smoothing Quadrilateral element with 20 DOF), whose performances in linear analysis have already been verified and demonstrated in references [22,23], for

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geometrically nonlinear analysis of composite plate and shell structures. The von-Karman's large deflection theory and the total Lagrangian (TL) approach are utilised in the small strain-large deformation formulation and then the solution of the nonlinear equilibrium equations is obtained by the arc-length method and automatic incremental algorithm [24]. With the aid of the assumed strain smoothing technique, the evaluations of the membrane, bending and geometric stiffness matrices are obtained via integration on the boundary of smoothing cells. This boundary integration contributes to the preservation of high accuracy of the method when highly distorted elements or coarse meshes are used. Numerical examples show that the present element is free from locking and exhibits good accuracy and stability in capturing geometric nonlinearity in composite plate/shell structures.

The remainder of the paper is outlined as follows. First, a brief review of the FSDT finite element formulations for geometrically nonlinear analysis is introduced in Section 2. The description of assumed strain smoothing approaches for the generalised strains and the tangent stiffness matrix of the element are derived in Section 3. Several numerical applications are carried out in Section 4 in order to verify and assess the performance of the proposed element. Finally, some concluding remarks are made in Section 5.

2. Finite element formulation for geometrically nonlinear analysis

2.1. Kinematic equations

Based on the FSDT, the laminated composite plate kinematics is governed by the midplane displacement $u_0,\ v_0,\ w_0$ and the rotation $\theta_x,\ \theta_y$ of the normal to the mid-surface about y- and x-axis, respectively [25]

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x, \\ v(x,y,z) &= v_0(x,y) + z\theta_y, \\ w(x,y,z) &= w_0(x,y). \end{aligned} \tag{1}$$

For large deformation analysis, the in-plane vector of Green-Lagrangian strain at any point in a plate element is

$$\boldsymbol{\epsilon} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} + \frac{1}{2}\left(u_{,x}^2 + v_{,x}^2 + w_{,x}^2\right) \\ v_{,y} + \frac{1}{2}\left(u_{,y}^2 + v_{,y}^2 + w_{,y}^2\right) \\ u_{,y} + v_{,x} + \left(u_{,x}u_{,y} + v_{,x}v_{,y} + w_{,x}w_{,y}\right). \end{Bmatrix} \tag{2}$$

Substituting Eq. (1) into Eq. (2) and considering the von Karman's large deflection assumption, the in-plane strain vector can be rewritten as

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_m + z\boldsymbol{\epsilon}_b, \tag{3}$$

in which

$$\boldsymbol{\epsilon}_b = \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{Bmatrix}, \tag{4}$$

$$\boldsymbol{\epsilon}_m = \begin{Bmatrix} u_{0,x} + \frac{1}{2}w_{,x}^2 \\ v_{0,y} + \frac{1}{2}w_{,y}^2 \\ u_{0,y} + v_{0,x} + w_{,x}w_{,y} \end{Bmatrix} = \underbrace{\begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}}_{linear part} + \underbrace{\begin{Bmatrix} \frac{1}{2}w_{,x}^2 \\ \frac{1}{2}w_{,y}^2 \\ w_{,x}w_{,y} \end{Bmatrix}}_{nonlinear part} \tag{5}$$

$$= \boldsymbol{\epsilon}_m^L + \boldsymbol{\epsilon}_m^{NL}. \tag{6}$$

The nonlinear term of the membrane strain–displacement vector can be rewritten as follows

$$\boldsymbol{\epsilon}_m^{NL} = \frac{1}{2}\underbrace{\begin{bmatrix} w_{,x} & 0 \\ 0 & w_{,y} \\ w_{,y} & w_{,x} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{Bmatrix} w_{,x} \\ w_{,y} \end{Bmatrix}}_{\boldsymbol{\theta}} = \frac{1}{2}\mathbf{H}\boldsymbol{\theta}, \tag{7}$$

in which $\boldsymbol{\theta}$ is termed the slope vector.

The transverse shear strain vector is given as

$$\boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \theta_x - w_{,x} \\ \theta_y - w_{,y} \end{Bmatrix}. \tag{8}$$

The constitutive relationship of laminated plates can be expressed as

$$\boldsymbol{\sigma}^* = \mathbf{D}^*\boldsymbol{\epsilon}^*, \tag{9}$$

where

$$\boldsymbol{\sigma}^* = \begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{T} \end{Bmatrix}, \quad \boldsymbol{\epsilon}^* = \begin{Bmatrix} \boldsymbol{\epsilon}_m \\ \boldsymbol{\epsilon}_b \\ \boldsymbol{\gamma} \end{Bmatrix}, \quad \mathbf{D}^* = \begin{bmatrix} \mathbf{A} & \mathbf{B} & 0 \\ \mathbf{B} & \mathbf{D} & 0 \\ 0 & 0 & \mathbf{C}_s \end{bmatrix}, \tag{10}$$

and $\mathbf{N} = [N_x\ N_y\ N_{xy}]$ is the in-plane traction resultant, $\mathbf{T} = [Q_x\ Q_y]$ is the out-of-plane traction resultant and $\mathbf{M} = [M_x\ M_y\ M_{xy}]$ is the out-of-plane moment resultant. $\mathbf{A}$ is the extensional stiffness, $\mathbf{D}$ is the bending stiffness, $\mathbf{B}$ is the bending-extension coupling stiffness and $\mathbf{C}_s$ is the transverse shear stiffness, respectively defined as

$$\begin{aligned} (A_{ij}, B_{ij}, D_{ij}) &= \int_{-h/2}^{h/2} (1, z, z^2)\bar{Q}_{ij}\,dz, \quad i,j = 1,2,6 \\ C_{sij} &= \int_{-h/2}^{h/2} \bar{Q}_{ij}\,dz, \quad i,j = 4,5 \end{aligned} \tag{11}$$

where $\bar{Q}_{ij}$ are the elastic constants with respect to the global x-axis and their detailed definitions can be found in reference [25].

2.2. Total Lagrangian finite element formulation

Consider a bounded domain $\Omega = \sum_{i=1}^{n_e}\Omega_i$ of a composite plate which is discretized into n_e finite elements. The finite element solution $\mathbf{u}$ of a displacement-based 4-node quadrilateral model is expressed as

$$\mathbf{u} = \begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix} = \sum_{i=1}^{4}\mathbf{N}_i\mathbf{q}_i, \tag{12}$$

where $\mathbf{N}_i$ is the bilinear shape function, $\mathbf{q}_{mi} = [u_i\ v_i]^T$, $\mathbf{q}_{bi} = [w_i\ \theta_{xi}\ \theta_{yi}]^T$ and $\mathbf{q}_i = \begin{bmatrix} \mathbf{q}_{mi} \\ \mathbf{q}_{bi} \end{bmatrix}$ are the displacement vectors of the element.

The total Lagrangian (TL) approach, in which the original configuration is taken as the reference, is usually used for geometrically nonlinear analysis. The finite element equation in the TL approach can be expressed in the following form

$${}^t\mathbf{K}_T\Delta q = {}^{t+\Delta t}\mathbf{P} - {}^t\mathbf{F}, \tag{13}$$

where ${}^t\mathbf{F}$ is the element internal force at time t, ${}^{t+\Delta t}\mathbf{P}$ is the element external force at time $t+\Delta t$, ${}^t\mathbf{K}_T$ is the element tangent stiffness matrix at time t and Δq is the element displacement increment.

The element tangent stiffness matrix $\mathbf{K}_T$ is defined as

$$\mathbf{K}_T = \mathbf{K}_L + \mathbf{K}_{NL} + \mathbf{K}_g, \tag{14}$$

where $\mathbf{K}_L$ represent the linear stiffness matrix, $\mathbf{K}_{NL}$ denotes the nonlinear stiffness matrix and $\mathbf{K}_g$ is the geometric stiffness matrix.

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