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Surface stress effect on the postbuckling and free vibrations of axisymmetric circular Mindlin nanoplates subject to various edge supports

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ABSTRACT

Postbuckling behavior of circular nanoplates and their free vibration characteristics in the vicinity of postbuckling domain are investigated with the consideration of surface stress effect. To this end, Mindlin's plate theory in conjunction with the Gurtin–Murdoch elasticity theory is utilized to derive nonlinear equations of motion incorporating geometric nonlinearity and surface stress effect. On the basis of generalized differential quadrature (GDQ) method, the non-classical governing differential equations are discretized along simply-supported and clamped boundary conditions and are then parameterized and solved using the pseudo arc-length continuation method. The postbuckling configurations of axisymmetric circular nanoplates are obtained as a function of applied axial compressive load based on the static analysis. Afterward, on the basis of dynamic analysis, the natural frequencies and associated mode-shapes of circular nanoplates corresponding to both prebuckling and postbuckling domains are predicted including surface stress effect. It is revealed that by decreasing the magnitude of thickness, the surface stress effect on the postbuckling configurations of nanoplates becomes more prominent. Moreover, the effect of surface stress may shift the postbuckling domain to higher or lower applied axial loads, depending on the magnitude and sign of surface elastic constants. These anticipations are the same corresponding to various edge supports.

1. Introduction

Thin plate structures have the capability to carry load after buckling. Therefore, the postbuckling behavior of these structures is completely essential in engineering analysis. There are various investigations in the literature in which buckling problems have been analyzed based on different analytical and numerical solutions to determine critical buckling loads and buckling mode-shapes. However, changes in geometry due to large displacement make the postbuckling analysis more complex that one needs to take geometric nonlinearity into account.

The fundamental nonlinear strain–displacement relations for a thin plate undergoing large deflection were obtained by von Karman in order to consider the stretching of the middle surface of the plate. According to the von Karman formula, numerous investigations were carried out to predict the postbuckling behavior of plates based on the general plate theory. Marguerre [1] developed an approximate postbuckling solution for an isotropic plate subjected to longitudinal compression. He proposed a new expression for Airy's stress function in terms of unknown coefficients. Later, Levy [2] derived more accurate solution on the basis of expanding the stress function in terms of independent Fourier series. He developed general expressions for the stress coefficients which can be then inserted in the nonlinear equilibrium equation to analyze the postbuckling problem. Afterwards, Coan [3] extended levy's work for the plates with stress free edges. By using the principle of virtual work, a closed-form solution to evaluate the initial postbuckling stiffness was conducted by Harris [4]. Feng [5] solved the postbuckling problem of plates by minimizing the potential energy in terms of three unknown displacement variables and considering nonlinear von Karman nonlinearity. Shin et al. [6] considered the postbuckling response of orthotropic laminates under uniform compression by developing a model based on Marguerre's methodology.

To mention some more recent works, Shen [7] presented postbuckling analysis for a simply-supported, shear deformable composite laminated plate subjected to combined axial and thermal loads. Wang [8] obtained the exact axisymmetric postbuckling equilibrium path for the circular plates subjected to uniform radial

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compression by the use of the power series method. The axisymmetric postbuckling behavior of polar orthotropic laminated moderately thick circular and annular plates under uniformly distributed in-plane radial compressive load at the outer edge was studied by Dumir et al. [9]. Liew et al. [10] examined the postbuckling response of functionally graded rectangular plates integrated with surface-bonded piezoelectric actuators and subjected to combined action of uniform temperature change, in-plane forces, and constant applied actuator voltage. A Ritz method based on kernel particle approximation for the field variables was proposed by Liew et al. [11] for the postbuckling analysis of laminated composite plates. Chen et al. [12] derived the postbuckling governing equations for the axisymmetric laminated circular plates with elliptical delamination. Recently, Ansari et al. [13] studied the axial postbuckling configurations of single-walled carbon nanotubes including thermal environment effect.

In recent years, the free vibration characteristics of structures around the postbuckling configuration have been an interesting subject for investigation. Emam and Nayfeh [14] presented an exact solution for the postbuckling configurations of composite beams and studied the vibrations that take place in the vicinity of a buckled equilibrium position. Xia et al. [15] obtained the nonlinear frequencies of a microbeam with initial lateral displacement due to buckled configurations. Recently, Rahimi et al. [16] studied the free vibration response of functionally graded beams in the vicinity of a buckled equilibrium position.

The mechanical behaviors of micro- and nanoscale structures are affected by various parameters known as size-effects. Hence, various size-dependent beam and plate models based on the modified continuum theories have been developed and applied to take the small scale effects into account [17–21]. Surface energy is one of these effects that is due to high surface to bulk ratio of the nanostructures and causes to exhibit different behavior compared to the conventional structures. To demonstrate the significant influence of surface stress effect on the elastic responses of nanostructures, the elastic continuum models based on Gurtin–Murdoch elasticity theory [22,23] has been widely applied for various micro- or nanoscale structures during the past several years. Miller and Shenoy [24] obtained the size-dependent results on the pure bending and unidirectional tension of nanobars and nanoplates, which were in excellent agreement with the atomistic simulation results by choosing proper material constants for the surface layer. Also, Shenoy [25] studied the size-dependent torsional rigidities of nanosized structural elements including surface effects on the basis of Gurtin–Murdoch elasticity theory. Sapsathiarn et al. [26] developed a beam model with considering the surface energy effects using the Gurtin–Murdoch theory. It was revealed that the developed model can predict the experimental results via size-independent properties such as bulk modulus and surface residual stress. The surface stress effect on the large deflection of ultra-thin films by incorporating Gurtin–Murdoch surface elasticity into the von Karman plate theory using the Hamilton's principle was investigated by Lim and He [27]. Ansari and Sahmani [28] proposed a non-classical solution to analyze bending and buckling behaviors of nanobeams incorporating surface stress effects corresponding to the different classical beam theories. Ansari and Sahmani [29] predicted the free vibration response of nanoplates including surface stress effects based on the continuum modeling approach. They implemented the Gurtin–Murdoch elasticity theory into different kinds of plate theory. Recently, Ansari et al. [30] studied the effect of surface stress on the free vibration characteristics of circular nanoplates subjected to various edge supports using the Gurtin–Murdoch elasticity theory and first-order shear deformation plate theory.

In the present investigation, on the basis of an efficient numerical solution methodology, the free vibration behavior of circular nanoplates around the postbuckling configuration is predicted incorporating surface effect. To this end, the Mindlin plate theory containing nonlinear von Karman relations in conjunction with Gurtin–Murdoch elasticity theory is utilized to develop a non-classical plate model including surface effect. After discretizing the size-dependent governing differential equations using the generalized differential quadrature (GDQ) method, the pseudo arc-length continuation method is employed to obtain the postbuckling configuration of axisymmetric circular nanoplates. Afterward, by using the solution of the nonlinear problem in the previous step, a dynamic analysis within the framework of an eigenvalue problem is performed to obtain the natural frequencies and associated mode-shapes of circular nanoplates corresponding to the both prebuckling and postbuckling domains including surface effects. Numerical results imply that the surface stress effect on the postbuckling configurations and free vibrations behavior in the vicinity of postbuckling configuration becomes more prominent when the magnitude of thickness reduces. Moreover, the effect of surface stress may shift the postbuckling domain to higher or lower applied axial loads, depending on the magnitude and sign of surface elastic constants.

2. Formulation of motion and corresponding boundary conditions

Consider a uniform circular nanoplate with the radius R, and thickness h, as shown in Fig. 1. It is assumed that the circular nanoplate is under uniform radial compression. The parameters λ and μ indicate the classical Lame constants while the surface Lame constants are shown by λ_s and μ_s. Also, the parameters ν, ρ, ρ_s and τ_s indicate the Poisson's ratio, mass density, surface density and surface residual stress, respectively. A cylindrical coordinate system (r, θ, z) is introduced at the one center of the mid-plane of the nanoplate, whereas the r axis is taken along the radius of the nanoplate, the θ axis in the tangential direction and the z axis is taken along the depth (thickness) direction. Also, the upper and lower surfaces of nanoplate at $z = \pm h/2$ are denoted by S^+ and S^-, respectively. The solid surfaces are regarded as layers without thickness which adhere to the underlying bulk material without slipping. Thus, the displacements are continuous. Based on the first-order shear deformation plate theory, in which the in-plane displacements are expanded as linear functions of the thickness coordinate and the transverse deflection is constant through the plate thickness and includes effects of shear deformation and rotary inertia, the displacement components (u_r, u_θ, u_z) along the axes (r, θ, z) for the axisymmetric problem can be written in a general form as

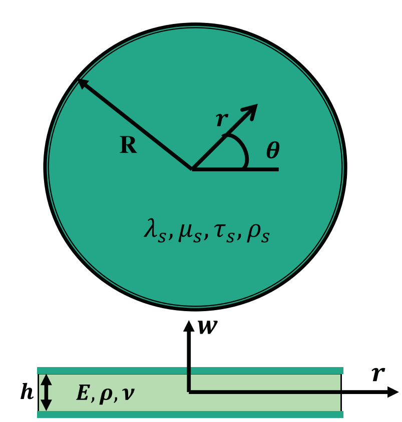

Fig. 1. Schematic of a circular nanoplate with upper and lower thin skin layers carrying surface effects: kinematic parameters, coordinate system and geometry.