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Systematic experimental and numerical study of bistable snap processes for anti-symmetric cylindrical shells

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ABSTRACT

Anti-symmetric cylindrical shell as a novel bistable composite structure, offers wide applications in many fields. The entire snap-through and snap-back processes of the anti-symmetric cylindrical shell are systematically studied through experimental investigation and numerical simulation. The experimental and numerical results are also compared with the analytical predictions. The parameters used to characterize the bistable performances of the shells, including coiled-up radii, stress distributions of the shell in the second stable state, and snap load are measured. Load–displacement curves and buckling phenomena in the snapping process are successfully captured. The influences of the geometrical sizes and layup conditions on the bistable performance of anti-symmetric cylindrical shells are discussed in detail. Comprehensive experimental and numerical results indicate that the initial mid-plane transverse radius and ply angle are two key factors that affect bistable behaviors in the same environmental conditions, which is accordant with theoretical predictions, whereas the number of plies and longitudinal length of the shell only influence on the snap load and stress distribution. The angle of embrace is demonstrated of no influence on bistable performance of anti-symmetric cylindrical shells.

1. Introduction

Smart composite structures, for example, the bistable composite structure [1–3], deployable composite structure [4–6] and composite lattice structure [7–10], etc., are ulitilized in many advanced engineering areas [11–13], for the favorable load-carrying capability, deformability and light-weight. Bistable composite structures have been proposed to develop morphing structures which finds its great potentials in aerospace and automotive industry [14–16]. In particular, bistable structures are able to maintain either of two stable configurations but without the need for a continuous power supply [17–20]. For this reason, it has been obtain an increasing attention from researchers around the world. Bistable composite structures can deform from one stable shape to another under an external stimulation induced by mechanical forces, piezoelectric patches, a varying temperature filed or SMAs (shape memory alloys) [21–24].

Bistable composite structures are usually manufactured from thin fiber reinforced laminates with different ply angles. As one of bistable composite structures possessing two stable cylindrical shapes, the cross-ply $[0_n/90_n]^T$ composite laminate has been investigated for over 30 years. The cross-ply $[0_n/90_n]^T$ laminate can be easily transformed from one stable state to another by an external load. A combined study based on the analytical, numerical and experimental methods conducted by Ren et al. [25] shows that the initial shape of a cross-ply composite laminate is affected by laminates' stacking sequence, the mold radius and thickness and size of laminates. Some analytical models based on high-order Ritz approximations and the parameter space explored via path-following techniques were presented by Pirrera et al. [26,27], in order to characterize structures' bistable performances and provide optimal design for multi-stable morphing structures. Shaw and Carrella [28] studied the snap processes of cross-ply composite laminates using numerical and experimental methods. The whole structure snapped from initial stable shape to second one under a movable force on the geometrical centroid. And the corresponding load–displacement curve for the snap process was also given. The sensitivity of material properties of the asymmetric laminates to environmental conditions (i.e. temperature) was also considered by some researchers. Moore et al. [29] examined the stability and thermal response of the asymmetric laminates using a combined method, the structure's transformations between two stable shapes were induced through a varying temperature field. Eckstein et al. [30] predicted the shapes of multi-stable laminated plates

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subjected to thermal loads by analytical method, with considering the thermal gradients and temperature-dependent material properties. Brampton et al. [31] demonstrated that bistable laminates are most sensitive to uncertainties in the material properties, ply thickness and the cure temperature change. It was shown that the material properties of laminates are highly sensitive to moisture absorption and temperature changes. Other related researches on cross-ply laminates can be found in Refs. [32–35].

The anti-symmetric composite cylindrical shell we emphasized in this paper, as another novel morphing structure with two stable cylindrical shapes, were discovered by Daton-Lovett [36] and then firstly studied by Iqbal et al. [37]. Such a composite structure is manufactured through using a cylindrical steel mold with the upper part pre-loaded. Based on the Classical Lamination Theory (CLT) and principle of minimum potential energy, Iqbal et al. [38] developed a theoretical model successfully to predict the coiled-up radius of the anti-symmetric shells. Galletly and Guest [39,40] then presented a beam model and a shell model which can be used to analyze the bistable behavior of the anti-symmetric cylindrical shell well although some deviations exist. Guest and Pellegrino [41] developed a simple two-parameter model which made the calculation of coiled-up radius of anti-symmetric shell easier. Norman et al. [42] investigated the multi-stable behaviors of pre-stressed corrugated shells. Zhang et al. [43] presented a new experimental method to capture the deformation process of an anti-symmetric shell. The required loads for a shell transforming between two stable configurations, i.e. snap-through and snap-back loads, were given experimentally. To the best knowledge of the authors, no previous work has emphasized the whole snap process of the anti-symmetric composite shell, including the deformation from initial shape to second one and its reverse process. There also has no systematic study on the bistable behaviors in the deformation process for anti-symmetric composite shells based on a combined method involving theoretical, numerical and experimental analyses. A thorough study of the snap-back process is beneficial for a better understanding of the bistable behaviors of anti-symmetric composite cylindrical shells.

Thus, the objective of this paper is to present a thorough study for the deformation process of anti-symmetric composite cylindrical shells using experimental and numerical methods as well as a theoretical analysis. Nine anti-symmetric cylindrical shell specimens made of T700-3234 carbon-fiber/epoxy prepreg are divided into five groups according the geometrical sizes and layup conditions to experimentally investigate the deformation processes and bistable behaviors in the processes. The snap-back process is firstly discussed in this paper through the combined method of experiments and finite element modeling. A comparison between analytical, experimental and numerical results is also given. Finally, the parameters influencing the bistable performances of anti-symmetric composite shells are discussed systematically based on the combined method. Comprehensive experimental and numerical results are provided to examine the effects of geometrical sizes (i.e. initial transverse radius, angle of embrace and longitudinal length) and layup conditions (i.e. ply angle, number of plies) on the bistable performances including the coiled-up radius, snap load and stress distribution at the second stable shape. It is believed that the results and findings presented in this paper are important and useful for design, manufacture and application of anti-symmetric bistable composite structures in aerospace industry.

2. Experimental investigation

Under external forces, a bistable anti-symmetric composite shell is able to snap between the two stable states: the initial shape and the second stable shape, which means the snapping is reversible. Consequently, the snap-through and snap-back processes utilized to describe the shell snapping from initial shape to the second one and its reverse process respectively are important for assessing the bistable performances of anti-symmetric composite cylindrical shells.

The anti-symmetric cylindrical shells are autoclave-manufactured in a cylindrical steel mold with a pre-load on the upper part using T700-3234 carbon-fiber/epoxy prepreg. The material properties are shown in Table 1. Numbered specimens with different ply angles, number of plies and geometrical sizes are listed in Table 2. Geometric parameters of the manufactured anti-symmetric composite shells are illustrated in Fig. 1: R denotes the mid-plane transverse radius of the first state, α is the ply angle, n indicates the number of plies, L defines the longitudinal length and γ is the angle of embrace. Two stable configurations of manufactured specimen No. 1 are shown in Fig. 2, as an example, its geometric parameters of first stable configuration are: $L = 100$ mm, $R = 25$ mm, $\gamma = 180^\circ$ and the layup is $[35^\circ/-35^\circ/35^\circ/-35^\circ]$.

The diagram of the two points loading method [43] is shown in Fig. 3. Forces are applied on the two straight edges' center along the z-direction to produce two external driving moments, the center of the shell is restricted in the z-direction, and the shell is supported by two plates. Along with the movement of the forces, the shell can transform from the first state to the second state.

Based on the experimental model, a testing machine (REGER3010) is used with the designed corresponding clamp and indenter assembled. The indenter provides the two forces and the clamp is used to support the specimens. As described above, the loads are applied in the center of the two straight edges until the shell snaps from one stable state to the other. The load–displacement curves and snap loads are recorded by the testing machine. Each specimen deforms several times between the two stable shapes under the same experimental condition so as to obtain steady testing data. Fig. 4 shows the whole snapping process of the shell in the experiment.

In order to compare the results from the experiments with theoretical analysis, the analytical solution of the coiled-up radii of the second state is introduced and derived here. The elastic behavior of a laminated plate element is characterized by its ***ABD*** matrix which correlates the mid-surface strains and curvatures to the corresponding stress resultants in the shell. The ***ABD*** matrix of a composite shell with anti-symmetric layup is as given below:

$$\begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{M} \end{bmatrix} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left[\begin{array}{ccc|ccc} A_{11} & A_{12} & 0 & 0 & 0 & B_{16} \\ A_{12} & A_{22} & 0 & 0 & 0 & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & 0 \\ \hline 0 & 0 & B_{16} & D_{11} & D_{12} & 0 \\ 0 & 0 & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & 0 & 0 & 0 & D_{66} \end{array}\right] \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{K} \end{bmatrix} \tag{1}$$

where ***N***, ***M*** are the external load and moment, respectively, ***A*** represents the extensional stiffness matrix, ***B*** is the coupling stiffness matrix and ***D*** denotes the bending stiffness matrix.

It can be found that coefficients D_{16} and D_{26} are zero, which means bending and twisting are decoupled, implying that the twist curvature $kxy = 0$. Since $\boldsymbol{B} \neq 0$ for the anti-symmetric laminate, the stretching and bending components are coupled in the

Table 1
Material properties of T700-3234 unidirectional lamina.

E_{11} (GPa)	E_{22} (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	t_{ply} (mm)
123	8.4	4.0	4.0	3.0	0.32	0.12