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# Dynamic analysis of composite laminated and sandwich hollow bodies of revolution based on three-dimensional elasticity theory

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ABSTRACT

This paper presents a semi-analytical procedure for solving linear vibration problems of composite laminated and sandwich hollow bodies of revolution with arbitrary combinations of boundary constraints in the framework of the three-dimensional theory of elasticity. Multilevel partitioning hierarchy, viz., multilayered body of revolution, individual layer and layer segment, is adopted in the theoretical analysis. The appropriate continuity constraints on common interfaces are imposed by means of a modified variational principle combined with the least-squares weighted residual method. The displacement field of each layer segment is characterized by a mixed series of basis functions, i.e., Fourier series and orthogonal polynomials. Numerical examples concerning the free vibrations of composite laminated and sandwich hollow cylinders, cones, and spheres, are presented to show the performance of the method, and comparisons of the present results are made with solutions available in the literature and those obtained from finite element analyses. With regard to the forced vibration problems, steady-state vibration responses of a sandwich hollow cylinder under a uniformly distributed normal harmonic pressure are analyzed, and time-domain solutions of composite laminated and sandwich hollow spheres subjected to various impulsive loads, including a rectangular pulse, a triangular pulse, a half-sine pulse and an exponential pulse, are also examined. Numerical experiments show that the present method is accurate, efficient and reliable for predicting the full spectrum of vibration behaviors of multilayered hollow bodies of revolution.



## 1. Introduction

Multilayered composite hollow bodies of revolution (e.g., cylinders, cones, spheres, ellipsoids and hyperboloids) are widely used as elementary structural components in modern aerospace structures, missiles, naval vehicles and other areas of engineering. These structural elements are commonly subjected to various forms of dynamic loadings in their service life and therefore, the knowledge of their vibration characteristics is of crucial importance from the standpoint of practical design. However, due to the complicated effects involving the transverse shear and normal deformations, coupled material behaviors and non-zero curvatures, etc., finding accurate vibration solutions for an arbitrarily thick multilayered hollow body of revolution with any kind of boundary conditions remains an extremely challenging task both analytically and numerically. The development of relevant theoretical methodologies and numerical modeling for multilayered bodies of revolution has thus received considerable attention from the research community. This paper is concerned with the theoretical development and associated applications of an efficient semi-analytical method for linear vibration analyses of composite laminated and sandwich hollow bodies of revolution based on the three-dimensional (3-D) theory of elasticity.

With respect to the structural modeling of multilayered hollow bodies of revolution, a preliminary distinction can be drawn between investigations relying on the 3-D elasticity formulations and theories based on the reduction of 3-D equations to two-dimensional (2-D) ones. In the case of thin and moderately thick composite hollow bodies of revolution (also called shells of revolution), the full 3-D vibration problems of these structures can be approximately characterized with 2-D multilayered shell theories, which reduce the dimensions of the problems by addressing the primary field variables of interest, such as displacements and stresses, in terms of certain averages over the thickness direction. These 2-D theories, in general, offer relatively simple mathematical manipulations in analytical or computational implementations, and are particularly suitable for the lower-frequency vibration

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predictions. Notable among these are the equivalent single-layer theories [1–4] (e.g., the classical theories, first- and high-order shear deformation theories), Zig-Zag models [5] and layer-wise theories [1], to name a few. For a comprehensive overview on these theories, interested readers may refer to the review works of Noor and Burton [6], Carrera [5,7] and Qatu [8]. In recent years, the advances in technology have allowed the transition of composite materials from secondary to primary structural components, and consequently thick multilayered bodies of revolution are usually demanded in engineering applications to sustain heavier loads. In such cases, the presence of the shear and normal deformations as well as the rotary inertia may reduce the effectiveness of the 2-D models since these effects are significant in thick multilayered bodies of revolution for both lower- and higher-order vibration modes. Moreover, in most of the 2-D theories, only part of the elastic constants are considered in the constitutive relations, which implies that the solutions will remain unchanged regardless of the variation of the values of the elastic constants not considered. Comparatively, no hypotheses about the distribution field of deformations and stresses are adopted in the elasticity theory, and contributions of all stresses and strains are considered by accounting for all the elastic constants. From a theoretical point of view, a 3-D analysis not only provides a full spectrum of vibration results for arbitrarily thick composite hollow bodies of revolution but also allows further physical insights, which cannot be predicted by reduced 2-D theories. Consequently, the analysis based on elasticity theory that takes into account all 3-D variations of stresses and strains is probably the only way to provide accurate responses for multilayered hollow bodies of revolution. This is particular true when the thickness dimension of a multilayered body of revolution becomes comparable to its principal radii of curvature. With this fact born in mind, the 3-D theory of elasticity is employed in the present analysis.

Despite the practical needs for 3-D elasticity solutions to multilayered hollow bodies of revolution, literatures on this topic are very scarce. Due to the intrinsic complexity of the problem based on the 3-D elasticity, exact solutions are not available for arbitrary thick multilayered bodies of revolution with general boundary conditions. Hence, most of the approaches for vibration problems of multilayered bodies of revolution are approximate in nature. In order to properly focus on the features and emphasis of the present paper, a brief review of the works pertaining to the 3-D dynamic analyses of multilayered bodies of revolution, including composite laminated and sandwich cylinders, cones and spheres, is reported below. Multilayered hollow cylinders have attracted more attention than any other shapes, not only due to their technical importance, but also because the mathematical theory is relatively simple. Srinivas [9] investigated the axisymmetric and flexural free vibrations of three-layered cross-ply hollow cylinders using the Frobenius's series method in the context of 3-D elasticity theory. However, solutions based on this method are found to be slowly convergent and the corresponding computations are quite sophisticated and tedious. Cheung and Wu [10] presented a finite layer method for predicting the free vibrations of thick laminated finite cylinders with various end conditions. By using the state-space method and dividing each physical layer into a number of fictitious layers, Ye and Soldatos [11] analyzed the 3-D free vibrations of cross-ply laminated hollow cylinders and cylindrical panels subjected to simply-supported boundary conditions. Later, they extended the state-space method to the vibration problems of cross-ply hollow cylinders having clamped edges [12] and composite cylinders composed of monoclinic elastic layers [13]. Ding and Tang [14] performed a 3-D free vibration analysis for thick laminated cylindrical shells with two clamped edges by means of the state-space method. Malekzadeh et al. [15] introduced a layerwise-differential method to derive the elasticity solutions for the free vibration problems of laminated hollow cylinders. Making use of the differential quadrature method in conjunction with the state-space technique, Alibeigloo [16] discussed the free vibration characteristics of anisotropic laminated cylindrical shells with several end conditions. Wu and Chen [17] carried out a free vibration analysis for simply-supported functionally graded piezoelectric sandwich cylinders using a modified Pagano method. Chen and Shen [18] presented a 3-D elasticity study for the free vibrations of a simply-supported cross-ply cylindrical shell containing a piezoelectric layer, in which the solution of the derived governing differential equations was obtained by the power series expansion method. The investigation of forced vibration behaviors, including steady-state and transient vibration responses, of multilayered cylinders plays an important role in structural design. However, very few theoretical studies have been reported on such kinds of problems. Kapuria and Kumari [19] analyzed the 3-D steady-state vibration responses of smart cross-ply circular cylindrical shells with simply-supported boundary conditions. The governing equations with variable coefficients were solved using the modified Frobenius's series method. The dynamic analyses of multilayered hollow cones based on the 3-D theory of elasticity are much more complex than those of cylinders, and therefore available solutions to this type of problem are rather limited. With regard to the free vibration problem, Wu and Wu [20] proposed an asymptotic differential quadrature approach to obtain the natural frequencies of laminated hollow cones with simply-supported boundary conditions. Based on the layerwise differential quadrature method, Talebitooti [21] investigated the free vibrations of thick, rotating laminated composite conical shells with different boundary conditions. It appears that the forced vibration analysis of multilayered hollow cones in the context of 3-D theory of elasticity has so far received no attention. Few studies concerning the 3-D vibration analyses of multilayered hollow spheres have been carried out. The radial vibrations of composite hollow spheres composed of spherically orthotropic layers were examined by Stavsky and Greenberg [22] using an exact method. Chen and coworkers [23,24] determined the free vibration behaviors of multilayered hollow spheres based on the state-space method. Concerning the forced vibration problems, Ding et al. [25] derived the elastodynamic solutions for multilayered hollow spheres by means of the method of separation of variables in conjunction with an orthotropic expansion technique. Later, this method was extended by Wang et al. [26] to the dynamic analysis of multilayered piezoelectric hollow spheres. The transient responses of two-layered elasto-piezoelectric hollow spheres were obtained by Wang and Ding [27], who employed the method of superposition combined with the state-space method to achieve the corresponding solutions.

During the above survey of the literature, the authors found that: (I) there exist a few studies concerning the free vibrations of multilayered bodies of revolution with particular geometrical configurations (e.g., cylindrical, conical and spherical shapes), but, to our knowledge, a general 3-D elasticity solution for a multilayered body of revolution with arbitrarily smooth shaped meridian does not seem to exist; (II) considerably less attention has been paid to the forced vibration problems, and in particular, rigorous analytical or numerical analyses for transient vibrations of multilayered bodies of revolution excited by arbitrary time-dependent forces have not been attempted so far; (III) limited sets of boundary conditions have been considered in previous research efforts. These apparent voids have thus formed the motivation of the present work. The focus of present investigation is to develop a computationally efficient methodology for linear vibration analyses of arbitrarily thick multilayered hollow bodies of revolution with any kind of boundary conditions. In the theoretical analysis, a multilayered hollow body of revolution is preliminarily divided into a number of individual layers along the locations of layer interfaces,