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Modelling of [0/90] laminates subject to thermal effects considering mechanical curvature and through-the-thickness strain

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ABSTRACT

Non-uniform curvatures and through-the-thickness strain have been incorporated in the Extended Classical Lamination Theory of Dano and Hyer to model curved shapes of bistable laminates. The indication of Tiersten about the use of the sine instead of the tangent function in the derivation of the bending curvature has been introduced. Two expansions of curvatures, named the mechanical and the mathematical curvatures, have been developed. Through-the-thickness strain, which is assumed to be uniform in thickness, has been also included in the analysis. Out-of-plane displacements of a grid of points marked at the deformed surface of a carbon/epoxy laminate of square shape have been experimentally obtained using a Coordinate Measuring Machine. Comparison between experimental results of displacements and different analytic approaches has been carried out.

1. Introduction

Cross-ply laminates have an anisotropic response due to elevated temperatures of manufacturing process, and residual thermal stresses lead to a curved shape at Room Temperature (RT) [1]. These laminates can have two stable states at RT: cylindrical state I with a major curvature in x axis and cylindrical state II with an opposite curvature in the orthogonal y axis, as shown in Fig. 1. One stable shape can change to the other by applying a small amount of energy. This is known as the snap-through phenomenon, making bi-stable laminates of interest for a wide range of engineering applications [2]. The asymmetric nature of the stacking sequence itself does not ensure bistability. When a laminate has a length-to-thickness ratio below a critical value, it only exists a single saddle-shaped solution [3]. When this ratio increases, the saddle solution becomes unstable and the stable solution bifurcates into two cylindrical shapes. That phenomenon occurs at bifurcation point. Therefore, the length-to-thickness ratio at the bifurcation point is one of the key parameters of the bistable behaviour of the laminate [4]. Loads and deformations had been monitored [5] through the bifurcation in the snap-through process of 2 ply [0/90]. It was seen that there were two closely coupled bifurcations. Light imperfections such as variations in thickness distribution and in material properties could be responsible for the lack of correlation between theory and experiments, particularly near the bifurcation point [6]. The elastic and thermoelastic properties that provided maximum curvatures of cross-ply plates have been studied [7] and a sensitivity analysis of uncertainties in material properties, geometry and environmental conditions has been recently carried out [8]. Otherwise, asymmetry in thickness could produce loss of bifurcation of thin [0/90] laminates [9].

Shapes of anti-symmetric laminates at RT do not conform the predictions of the Classical Lamination Theory (CLT) [1], which is a geometrically linear theory. When geometrical nonlinearities are neglected the occurrence of multiple deformed shapes cannot be predicted. Hyer was able to simulate the bi-stable behaviour of [0/90] plates by taking into account geometrical nonlinearities of von Kármán [10] within the CLT. The original model of Hyer was based on the Rayleigh–Ritz method which consists in searching approximate displacement solutions, satisfying kinematics boundary conditions. This procedure involved the employment of moderate order polynomial functions for the description of the displacement [6,11,12]. The predictions of the Hyer's original model are shown in agreement with experiments on [0/90] square plates and Finite Element Models (FEM) [13]. Other contributions based on displacement assumptions came from Jun and Hong [3,14].

The displacement functions were used to obtain the strains needed to compute the laminate strain energy. Dano and Hyer [15] used directly for the first time approximations for the laminate mid-plane strains. Initially they used a set of complete polynomials with 28 unknown coefficients, but then they realized that the use of 14 coefficients was enough [15–17]. This approach is named Extended Classical Lamination Theory (ECLT). Based on the ECLT, bi-stable laminate design for a reversible snap-through

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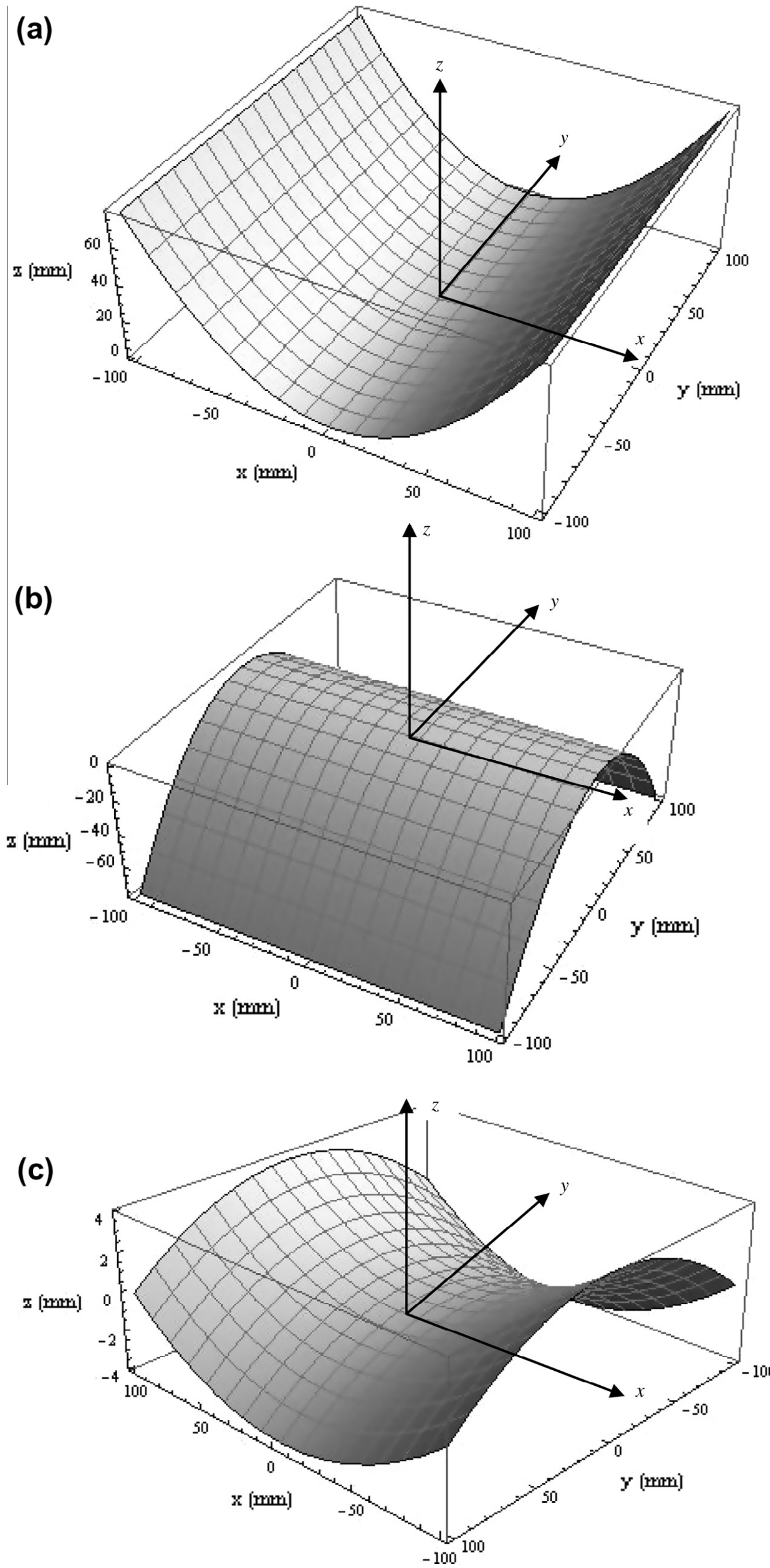

Fig. 1. Room temperature shapes of a square $[0/90]_T$ laminate: (a) stable cylindrical shape; (b) opposite cylindrical shape; and (c) unstable saddle shape.

in adaptive structures has recently been studied [18] reducing the number of unknown coefficients based on ply design, and an analytical approach has been presented [19]. Difficulties were encountered in simulating multistable/bifurcation behaviour by FEM solvers [9]. Moreover, the simulation time for such class of problems can be very high, depending on the adopted level of