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Mass optimization of functionally graded plate for mechanical loading in the presence of deflection and stress constraints

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ABSTRACT

In this work, a method for the single-objective optimization of material distribution of simply supported functionally graded isotropic plates is presented. The material composition is assumed to vary only in the thickness direction. Piecewise cubic interpolation of volume fractions are used to calculate volume fractions of constituent material phases at a point; these fractions are defined at a limited number of evenly spaced control points. The effective material properties of the plate are obtained by applying linear rule of mixtures. Behavior of functionally graded plate is predicted by the assumptions of the third-order shear deformation theory of Reddy. Exact solutions for deflections and stresses of simply supported plates are presented by using Navier type solution technique. Those volume fractions at control points that are selected as decision variables are optimized by two evolutionary algorithms: (1) Real-coded genetic algorithm and (2) particle swarm optimization algorithm. Three models are optimized as primary goal to verify the capability and efficiency of the proposed model with flexibility and stress constraints under various transverse loads. Secondary goal of this article is survey accuracy and convergence of two algorithms aforementioned. The proposed framework for designing functionally graded plates under pure mechanical conditions has been furnished by the founded results.

1. Introduction

Functionally Graded Materials (FGMs) as advanced materials from family of engineering composites are made of two or more constituent phases and have continuous and smoothly varying composition. These advanced materials which have engineered gradients of composition, structure and/or specific properties in the preferred direction/orientation are superior to the homogeneous materials which are composed of similar constituents [1]. Designing objects with Functionally Graded Material (FGM) distributions has been the subject of substantial research interest in recent years. This popularity is largely due to the many excellent and unique properties that FGM objects possess: for instance, the combined advantages of different materials, improved material compatibilities and good adaptability to versatile working conditions [2].

In the past few decades, representation and modeling of variation of FGM's volume fraction have been based on one of the following approaches; (1) power-law [3–11], (2) exponential function [3,12,13], and (3) sigmoid functions [3], trigonometric function [14,15], step-formed [16,17] and polynomial functions [2,18–20]. Analysis of FG plates in the past have been based on one of the following approaches; (1) classical laminated plate theory (CLPT) [3,21], firs-order shear deformation theory (FSDT) [6] and higher-order shear deformation theories (HSDT) [4,5,7,9,11,14,22–25] and (2) three-dimensional elasticity theory, quasi-3D shear deformation theory and layerwise theory [12,13,26,27].

It is worth mentioning that performance of an FGM is not only a function of properties of its constituents phases, but is also directly associated to ability of a designer in using materials in the most optimal manner. Therefore, optimizing material distribution is a crucial step for designing functionally graded structures. In the context of optimization of functionally graded materials, much less research has been done to analysis of these materials. In addition, most of these researches were founded on thermo-mechanical conditions [2,8,10,16,17,19,20,28,29] rather than mechanical conditions [9,15,27,30,31].

Magnucka-Blandzi and Magnucki [15] presented the optimal dimensionless parameters of a simply supported sandwich beam with a metal foam core under a uniformly distributed transverse load or a compressive axially force. The effective design was consists in achieving maximum of critical force of the axially compressed sandwich beam or minimum of mass of the structure. The distributions of properties across the thickness (core) and in

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the plane (face sheets) that minimize the interlaminar stresses at the interface with the core have been presented by Icardi and Ferrero [27]. The bending stiffness was maximized, while the energy due to interlaminar stresses is minimized. Qian and Batra [9] analyzed a bi-directional FG plate for optimal natural frequencies. They used genetic algorithm to optimize the spatial volume fractions of the two constituents so as to maximize either the first or the second natural frequency of the structure. Lipton [30] introduced a methodology for the design of structural components made from composite materials in the presence of stress constraints. A steepest decent method was developed for finding functionally graded materials that provide the maximum torsional rigidity while keeping the mean square stress inside the composite structure below a prescribed level. Optimal designs of an FGM dental implant have been presented by Sadollah and Bahreininejad [31]. Metaheuristic algorithms such as the genetic algorithms (GAs) and simulated annealing (SA) have been adopted to develop a multi-objective optimal design for FGM implantation design. These materials are now developed for general use as structural components. Thus there is need to provide comprehensive design optimization in the field of functionally graded materials under purely mechanical conditions.

In this article, a method was proposed for optimizing material distribution of simply supported functionally graded plates, which was performed through demonstrating volume fraction variation of an isotropic FGM plate using piecewise cubic Hermite polynomials. Volume fraction at a point is calculated via range-restricted interpolation of volume fractions at a limited number of control points which are equally spaced in thickness direction [19]. The volume fractions at control points are selected as decision variables because they can be changed to have different continuous volume fraction profiles. Behavior of FG plate is predicted by the assumptions of the third-order shear deformation theory of Reddy [7,32]. Necessary equilibrium equations and boundary conditions are derived by employing the principle of virtual work. Exact solutions for deflections and stresses of simply supported plates are presented by using Navier type solution technique. The volume fraction profile is optimized using two single-objective evolutionary algorithms: (1) Real-coded genetic algorithm (RGA) and (2) particle swarm optimization algorithm (PSO). Three models are optimized as primary goal to verify the capability and efficiency of the proposed model with flexibility and stress constraints under various transverse loads. Secondary goal of this article is survey accuracy and convergence of two algorithms aforementioned. The present method is able to deal with any kind of loading that could be expanded to a trigonometric series.

This article is organized as follows. In Section 2, volume fraction's range-restricted piecewise cubic interpolation, the estimation of effective material properties, the formulation and trigonometric-series analytical solution of the FG plate are discussed. Details about the single-objective optimization algorithms are given in Section 3. In Section 4, the proposed method is utilized for analyzing and optimizing materials distribution for three model problems. The proposed framework for designing functionally graded plates under pure mechanical conditions has been furnished by the founded results.

2. Problem formulation

Consider a solid rectangular plate of length a, width b and thickness h made of functionally graded material. In Rectangular Cartesian coordinates x, y, z, the functionally graded plate occupies the region $[0, a] \times [0, b] \times [-h/2, h/2]$ respectively. It is assumed that the material properties of the plate vary only in the thickness direction (Fig. 1).

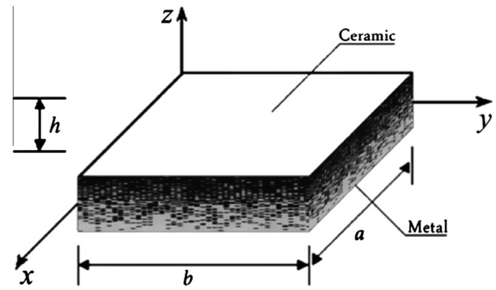

Fig. 1. Geometry of rectangular functionally graded plate.

2.1. Volume fraction distribution

Optimizing material distribution of a two-phase functionally graded material is equivalent to optimizing volume fraction profile $V(z)$ of one of its constituent phases. The present formulation applied a limited number of control points in thickness direction so as to decrease the number of design variables. Volume fraction of any thickness location was calculated using piecewise cubic interpolation by the volume fractions at control points. Therefore, volume fraction profile in thickness direction of the plate could be obtained by volume fraction values at control points [19].

A sum of $N+1$ equally spaced control points are used in thickness direction at locations $z_n = z_{bottom} + (z_{top} - z_{bottom})(n-1)/N$, where $n = 1, \ldots, N+1$. V_n represents the volume fraction at the control point located at z_n (Fig. 2). In the present formulation, volume fractions $V_1, \ldots, V_{N+1}$ at the control points are considered decision variables. Physical constraints of the problem require the interpolated volume fraction to be strictly within the range of 0–1 at all points within the domain, i.e. $0 \leqslant V(z) \leqslant 1$. For volume fraction modeling, a range restricted piecewise cubic interpolation is utilized to obtain smooth material composition profiles. Restriction of the interpolant for remaining between the desired bounding values is obtained through setting upper and lower limits on the slope of the interpolated volume fraction distribution at the control points. Volume fraction distribution $V(z)$ within the interval $z \in [z_n, z_{n+1}]$ is interpolated in the following way:

$$V(z) = V_n H_1(z) + S_n H_2(z) + V_{n+1} H_3(z) + S_{n+1} H_4(z), \tag{1}$$

where volume fraction and slope of volume fraction distribution are indicated by V_n and S_n at the control point z_n, respectively. Definition of Hermite basis functions $H_k(z)$ [19] are stated as:

$$\begin{aligned} H_1(z) &= \frac{2}{\bar{h}^3}\left(z - z_n + \frac{\bar{h}}{2}\right)(z - z_{n+1})^2, \\ H_2(z) &= \frac{1}{\bar{h}^2}(z - z_n)(z - z_{n+1})^2, \\ H_3(z) &= -\frac{2}{\bar{h}^3}(z - z_n)^2\left(z - z_{n+1} - \frac{\bar{h}}{2}\right), \\ H_4(z) &= \frac{1}{\bar{h}^2}(z - z_n)^2(z - z_{n+1}), \end{aligned} \tag{2}$$

where $\bar{h} = z_{n+1} - z_n = (h_{top} - h_{bottom})/N$. 3-point centered finite difference formula is used to estimate the slopes,

$$S_n = (V_{n+1} - V_{n-1})/2\bar{h} \quad \text{for } n = 2, 3, \ldots, N. \tag{3}$$

Slopes at the bottom and top surfaces of the plate are assessed using one sided finite difference formula,