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Nonlinear free vibration analysis of functionally graded third-order shear deformable microbeams based on the modified strain gradient elasticity theory

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ABSTRACT

In the present investigation, a numerical analysis is conducted to predict size-dependent nonlinear free vibration characteristics of third-order shear deformable microbeams made of functionally graded materials (FGMs). For this purpose, the modified strain gradient elasticity theory and von Karman geometric nonlinearity are implemented into the classical third-order shear deformation beam theory to develop a nonclassical higher-order beam model including three additional length scale parameters to capture size effect efficiently. It is assumed that the material properties of the FGM microbeams are evaluated by the Mori–Tanaka homogenization technique. On the basis of the Hamilton's principle, the size-dependent nonlinear governing differential equations of motion and associated boundary conditions are derived and then discretized along various end supports by employing generalized differential quadrature (GDQ) method. A direct iterative process corresponding to both positive and negative deflection cycles is adopted. Secondly, a parametric study is performed to demonstrate the influences of the values of dimensionless length scale parameter, material property gradient index and length to thickness aspect ratio on the linear and nonlinear natural frequencies of FGM microbeams.

1. Introduction

The rapidly developing nanotechnology leads to microbeams have been widely used in Micro- and Nano-Electro-Mechanical Systems (MEMS and NEMS) in which the thickness of beams is typically on the order of microns and sub-microns [1–4]. It has been experimentally shown that for various materials such as metals, polymers or functionally graded materials (FGMs), by closing the thickness of structures to internal material length scale parameter, microstructural effects exist which have essential influence on mechanical properties and deformation characteristics [5–7].

The well-known classical continuum theories, which do not account for such size effects because of the lack of additional length scale parameter, are inadequate to predict size-dependent responses of microscale structures. Therefore, needs exist to develop size-dependent models to estimate microstructural size dependency of these structures. To this end, several nonclassical higher-order continuum theories have been received increasing attention in modeling small sized structures.

One of the size-dependent continuum theories is strain gradient theory, introduced by Fleck and Hutchinson [8], which can be named as an extension of the Mindlin's theory [9]. After that, Lam et al. [5] proposed a modified strain gradient elasticity theory in which a new additional equilibrium equations to govern the characteristics of higher-order stresses including three material length scale parameters. In recent years, various studied have been conducted to analyze size-dependent behaviors of microstructures. For instance, Zhao et al. [10] analyzed the nonlinear static bending deformation, the postbuckling problem and the nonlinear vibration of Euler–Bernoulli microbeams based on the modified strain gradient elasticity theory. Recently, Zhang et al. [11] developed a novel size-dependent FGM curved microbeams based on the modified strain gradient theory and *n* shear deformation theory. Akgoz and Civalek [12–14] employed strain gradient and modified couple stress elasticity theories to study bending, buckling and free vibration of Euler–Bernoulli microbeams. More recently, nonlinear forced vibrations of a microbeam employing the strain gradient theory were investigated by Ghayesh et al. [15]. Mahammadi and Mahzoon [16] studied thermal effects on postbuckling of nonlinear microbeams using the modified strain gradient elasticity theory. Ansari et al. [17] investigated the free vibration characteristics of FGM microbeams based on strain gradient Timoshenko beam theory. Also, Ansari et al. [18] studied bending, buckling and free

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vibration responses of FGM Timoshenko microbeams based on strain gradient elasticity theory. The effect of small scale on the nonlinear free vibration behavior of FGM Timoshenko microbeams was predicted by Ansari et al. [19] on the basis of strain gradient theory. Sahmani and Ansari [20] presented recently the prediction of buckling behavior of size-dependent FGM third-order shear deformable microbeams including thermal environment effect using modified strain gradient elasticity theory.

Furthermore, size-dependent models for other types of FGM microstructures such as microplates and microshells have been developed. For example, Sahmani and Ansari [21] studied the free vibration behavior of FGM higher-order shear deformable microplates based on strain gradient theory. Recently, on the basis of modified couple stress theory, Sahmani et al. [22] predicted dynamic stability response of FGM higher-order shear deformable microshells.

The objective of the current study is to establish a nonlinear third-order shear deformation microbeam model based on the modified strain gradient elasticity theory. This developed nonclassical beam model contains three additional material length scale parameters to capture the microstructural effect. A simple power law function is utilized to model the through-thickness variation in the material properties of the FGM microbeams. By using Hamilton's principle, the nonlinear equations of motion together with corresponding boundary conditions are derived. Afterward, by employing generalized differential quadrature (GDQ) method, the higher-order equations of motion are discretized along with various types of end supports to calculate linear and nonlinear natural frequencies of FGM microbeams. A detailed parametric study is presented to indicate the influences of length scale parameter, gradient index and length to thickness aspect ratio on the nonlinear vibration behavior of FGM third-order shear deformable microbeams.

2. Homogenization of material properties

An initially straight FGM microbeam of length L between two immovable supports and thickness h that is made from a mixture of ceramics and metals is considered as sketched in Fig. 1. It is assumed that the materials at bottom surface $(z = -h/2)$ and top surface $(z = h/2)$ of the microbeam are ceramics and metals, respectively. In the figure, the axial body force per unit length, the transverse distributed force intensity per unit length and the body couples imposed on the sections as couple per unit axial length in the y-direction are denoted by $G(x,t)$, $F(x,t)$ and $C(x,t)$, respectively. The local effective material properties of the FGM microbeam can be calculated using homogenization method that is based on the Mori–Tanaka scheme. On the basis of Mori–Tanaka homogenization technique, the local effective bulk modulus K_e and shear modulus μ_e can be estimated as [23–25]

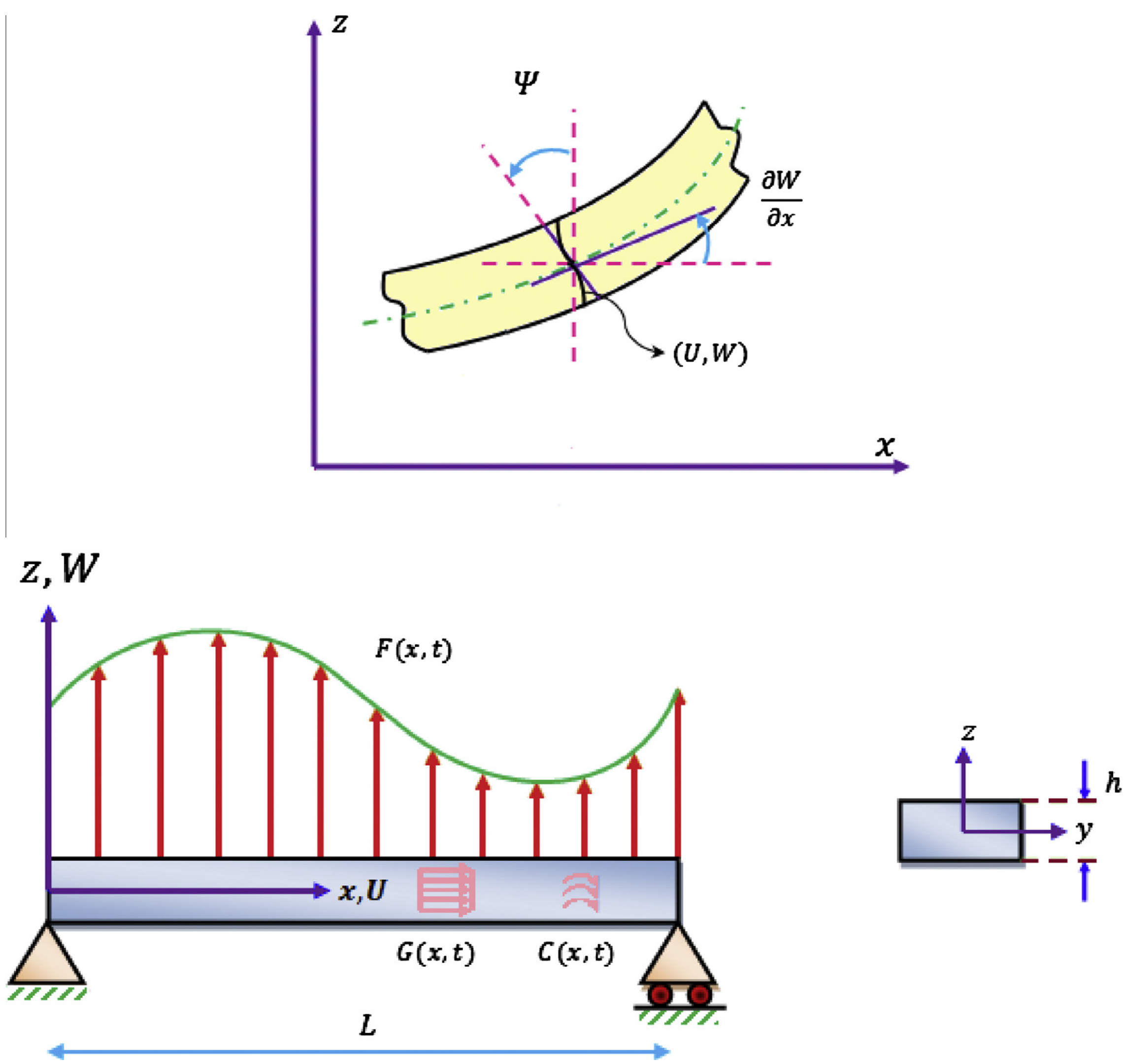

Fig. 1. Schematic of an FGM third-order shear deformable microbeam: kinematic parameters, coordinate system, geometry and loading.